Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Derivatives

The Definition of the Derivative

1. Use the definition of the derivative to find the derivative of,

$$f(x) = 6$$

Solution
There really isn't much to do for this problem other than to plug the function into the definition of the derivative and do a little algebra.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{6-6}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$$

So, the derivative for this function is,

$$\boxed{f'(x) = 0}$$

2. Use the definition of the derivative to find the derivative of,

$$V\left(t\right) = 3 - 14t$$

Step 1

First we need to plug the function into the definition of the derivative.

$$V'\left(t\right) = \lim_{h \to 0} \frac{V\left(t+h\right) - V\left(t\right)}{h} = \lim_{h \to 0} \frac{3 - 14\left(t+h\right) - \left(3 - 14t\right)}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also watch for the parenthesis on the second function evaluation. You are subtracting off the whole function and so you need to make sure that you deal with the minus sign properly. Either put in the parenthesis as we've done here or make sure the minus sign get distributed through properly. This is another very common error and one that if you make will often make the problem impossible to complete.

Step 2

Now all that we need to do is some quick algebra and we'll be done.

$$V'\left(t\right) = \lim_{h \to 0} \frac{3 - 14t - 14h - 3 + 14t}{h} = \lim_{h \to 0} \frac{-14h}{h} = \lim_{h \to 0} \left(-14\right) = -14$$

The derivative for this function is then,

$$\boxed{V'\left(t\right) = -14}$$

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3. Use the definition of the derivative to find the derivative of,

$$g\left(x\right) = x^2$$

Step 1

First we need to plug the function into the definition of the derivative.

$$g'\left(x\right) = \lim_{h \to 0} \frac{g\left(x+h\right) - g\left(x\right)}{h} = \lim_{h \to 0} \frac{\left(x+h\right)^2 - x^2}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Step 2

Now all that we need to do is some quick algebra and we'll be done.

$$g'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0}(2x+h) = 2x$$

The derivative for this function is then,

$$\boxed{g'(x) = 2x}$$

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4. Use the definition of the derivative to find the derivative of,

$$Q(t) = 10 + 5t - t^2$$

Step 1

First we need to plug the function into the definition of the derivative.

$$Q'(t) = \lim_{h \to 0} \frac{Q(t+h) - Q(t)}{h} = \lim_{h \to 0} \frac{10 + 5(t+h) - (t+h)^2 - (10 + 5t - t^2)}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also watch for the parenthesis on the second function evaluation. You are subtracting off the whole function and so you need to make sure that you deal with the minus sign properly. Either put in the parenthesis as we've done here or make sure the minus sign get distributed through properly. This is another very common error and one that if you make will often make the problem impossible to complete.

Step 2

Now all that we need to do is some algebra (and it might get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$\begin{aligned} Q'(t) &= \lim_{h \to 0} \frac{10 + 5t + 5h - t^2 - 2th - h^2 - 10 - 5t + t^2}{h} \\ &= \lim_{h \to 0} \frac{h(5 - 2t - h)}{h} = \lim_{h \to 0}(5 - 2t - h) = 5 - 2t \end{aligned}$$

The derivative for this function is then,

$$\boxed{Q'(t) = 5 - 2t}$$

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5. Use the definition of the derivative to find the derivative of,

$$W(z) = 4z^2 - 9z$$

Step 1
First we need to plug the function into the definition of the derivative.

$$W'(z) = \lim_{h \to 0} \frac{W(z+h) - W(z)}{h} = \lim_{h \to 0} \frac{4(z+h)^2 - 9(z+h) - \left(4z^2 - 9z\right)}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also watch for the parenthesis on the second function evaluation. You are subtracting off the whole function and so you need to make sure that you deal with the minus sign properly. Either put in the parenthesis as we've done here or make sure the minus sign get distributed through properly. This is another very common error and one that if you make will often make the problem impossible to complete.

Step 2
Now all that we need to do is some algebra (and it might get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$\begin{aligned} W'(z) &= \lim_{h \to 0} \frac{4\left(z^2 + 2zh + h^2\right) - 9z - 9h - 4z^2 + 9z}{h} \\ &= \lim_{h \to 0} \frac{4z^2 + 8zh + 4h^2 - 9z - 9h - 4z^2 + 9z}{h} \\ &= \lim_{h \to 0} \frac{h(8z + 4h - 9)}{h} = \lim_{h \to 0}(8z + 4h - 9) = 8z - 9 \end{aligned}$$

The derivative for this function is then,

$$\boxed{W'(z) = 8z - 9}$$

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6. Use the definition of the derivative to find the derivative of,

$$f(x) = 2x^3 - 1$$

Step 1
First we need to plug the function into the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^3 - 1 - \left(2x^3 - 1\right)}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also watch for the parenthesis on the second function evaluation. You are subtracting off the whole function and so you need to make sure that you deal with the minus sign properly. Either put in the parenthesis as we've done here or make sure the minus sign get distributed through properly. This is another very common error and one that if you make will often make the problem impossible to complete.

Step 2
Now all that we need to do is some algebra (and it might get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{2\left(x^3+3x^2h+3xh^2+h^3\right)-1-2x^3+1}{h} \\ &= \lim_{h\to 0} \frac{2x^3+6x^2h+6xh^2+2h^3-1-2x^3+1}{h} \\ &= \lim_{h\to 0} \frac{h\left(6x^2+6xh+2h^2\right)}{h} = \lim_{h\to 0}\left(6x^2+6xh+2h^2\right) = 6x^2 \end{aligned}$$

The derivative for this function is then,

$$\boxed{f'(x) = 6x^2}$$

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7. Use the definition of the derivative to find the derivative of,

$$g(x) = x^3 - 2x^2 + x - 1$$

Step 1
First we need to plug the function into the definition of the derivative.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - 2(x+h)^2 + x + h - 1 - \left(x^3-2x^2+x-1\right)}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also watch for the parenthesis on the second function evaluation. You are subtracting off the whole function and so you need to make sure that you deal with the minus sign properly. Either put in the parenthesis as we've done here or make sure the minus sign get distributed through properly. This is another very common error and one that if you make will often make the problem impossible to complete.

Step 2
Now all that we need to do is some algebra (and it will get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-2\left(x^2+2xh+h^2\right)+x+h-1-\left(x^3-2x^2+x-1\right)}{h} \\ &= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-2x^2-4xh-2h^2+x+h-1-x^3+2x^2-x+1}{h} \\ &= \lim_{h\to 0} \frac{h\left(3x^2+3xh+h^2-4x-2h+1\right)}{h} \\ &= \lim_{h\to 0}\left(3x^2+3xh+h^2-4x-2h+1\right) = 3x^2-4x+1 \end{aligned}$$

The derivative for this function is then,

$$\boxed{g'(x) = 3x^2-4x+1}$$

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8. Use the definition of the derivative to find the derivative of,

$$R(z) = \frac{5}{z}$$

Step 1
First we need to plug the function into the definition of the derivative.

$$R'(z) = \lim_{h\to 0}\frac{R(z+h)-R(z)}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{5}{z+h}-\frac{5}{z}\right)$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also note that in order to make the problem a little easier to read rewrote the rational expression in the definition a little bit. This doesn't need to be done, but will make things a little nicer to look at.

Step 2
Next we need to combine the two rational expressions into a single rational expression.

$$R'(z) = \lim_{h\to 0}\frac{1}{h}\left(\frac{5z-5(z+h)}{z(z+h)}\right)$$

Step 3
Now all that we need to do is some algebra and we'll be done.

$$R'(z) = \lim_{h \to 0} \frac{1}{h}\left(\frac{5z - 5z - 5h}{z(z+h)}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{-5h}{z(z+h)}\right) = \lim_{h \to 0} \frac{-5}{z(z+h)} = -\frac{5}{z^2}$$

The derivative for this function is then,

$$\boxed{R'(z) = -\frac{5}{z^2}}$$

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9. Use the definition of the derivative to find the derivative of,

$$V(t) = \frac{t+1}{t+4}$$

Step 1
First we need to plug the function into the definition of the derivative.

$$V'(t) = \lim_{h \to 0} \frac{V(t+h) - V(t)}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{t+h+1}{t+h+4} - \frac{t+1}{t+4}\right)$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Also note that in order to make the problem a little easier to read rewrote the rational expression in the definition a little bit. This doesn't need to be done, but will make things a little nicer to look at.

Step 2
Next we need to combine the two rational expressions into a single rational expression.

$$V'(t) = \lim_{h \to 0} \frac{1}{h}\left(\frac{(t+h+1)(t+4) - (t+1)(t+h+4)}{(t+h+4)(t+4)}\right)$$

Step 3
Now all that we need to do is some algebra (and it will get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$V'(t) = \lim_{h \to 0} \frac{1}{h}\left(\frac{t^2 + th + 5t + 4h + 4 - \left(t^2 + th + 5t + h + 4\right)}{(t+h+4)(t+4)}\right)$$
$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{t^2 + th + 5t + 4h + 4 - t^2 - th - 5t - h - 4}{(t+h+4)(t+4)}\right)$$
$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{3h}{(t+h+4)(t+4)}\right) = \lim_{h \to 0} \frac{3}{(t+h+4)(t+4)} = \frac{3}{(t+4)^2}$$

The derivative for this function is then,

$$\boxed{V'(t) = \frac{3}{(t+4)^2}}$$

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10. Use the definition of the derivative to find the derivative of,

$$Z(t) = \sqrt{3t-4}$$

Step 1
First we need to plug the function into the definition of the derivative.

$$Z'(t) = \lim_{h \to 0} \frac{Z(t+h) - Z(t)}{h} = \lim_{h \to 0} \frac{\sqrt{3(t+h)-4} - \sqrt{3t-4}}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Step 2
Next we need to rationalize the numerator.

$$Z'(t) = \lim_{h \to 0} \frac{\left(\sqrt{3(t+h)-4} - \sqrt{3t-4}\right)}{h} \frac{\left(\sqrt{3(t+h)-4} + \sqrt{3t-4}\right)}{\left(\sqrt{3(t+h)-4} + \sqrt{3t-4}\right)}$$

Step 3
Now all that we need to do is some algebra (and it will get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$Z'(t) = \lim_{h\to 0} \frac{3(t+h)-4-(3t-4)}{h\left(\sqrt{3(t+h)-4}+\sqrt{3t-4}\right)} = \lim_{h\to 0} \frac{3t+3h-4-3t+4}{h\left(\sqrt{3(t+h)-4}+\sqrt{3t-4}\right)}$$

$$= \lim_{h\to 0} \frac{3h}{h\left(\sqrt{3(t+h)-4}+\sqrt{3t-4}\right)} = \lim_{h\to 0} \frac{3}{\sqrt{3(t+h)-4}+\sqrt{3t-4}} = \frac{3}{2\sqrt{3t-4}}$$

Be careful when multiplying out the numerator here. It is easy to lose track of the minus sign (or parenthesis for that matter) on the second term. This is a very common mistake that students make.

The derivative for this function is then,

$$\boxed{Z'(t) = \frac{3}{2\sqrt{3t-4}}}$$

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11. Use the definition of the derivative to find the derivative of,

$$f(x) = \sqrt{1-9x}$$

Step 1
First we need to plug the function into the definition of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1-9(x+h)}-\sqrt{1-9x}}{h}$$

Make sure that you properly evaluate the first function evaluation. This is one of the more common errors that students make with these problems.

Step 2
Next we need to rationalize the numerator.

$$f'(x) = \lim_{h\to 0} \frac{\left(\sqrt{1-9(x+h)}-\sqrt{1-9x}\right)}{h} \frac{\left(\sqrt{1-9(x+h)}+\sqrt{1-9x}\right)}{\left(\sqrt{1-9(x+h)}+\sqrt{1-9x}\right)}$$

Step 3
Now all that we need to do is some algebra (and it will get a little messy here, but that is somewhat common with these types of problems) and we'll be done.

$$f'(x) = \lim_{h\to 0} \frac{1-9(x+h)-(1-9x)}{h\left(\sqrt{1-9(x+h)}+\sqrt{1-9x}\right)} = \lim_{h\to 0} \frac{1-9x-9h-1+9x}{h\left(\sqrt{1-9(x+h)}+\sqrt{1-9x}\right)}$$

$$= \lim_{h\to 0} \frac{-9h}{h\left(\sqrt{1-9(x+h)}+\sqrt{1-9x}\right)} = \lim_{h\to 0} \frac{-9}{\sqrt{1-9(x+h)}+\sqrt{1-9x}} = \frac{-9}{2\sqrt{1-9x}}$$

Be careful when multiplying out the numerator here. It is easy to lose track of the minus sign (or parenthesis for that matter) on the second term. This is a very common mistake that students make.

The derivative for this function is then,

$$\boxed{f'(x) = \frac{-9}{2\sqrt{1-9x}}}$$