Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Computing Definite Integrals

1. Evaluate each of the following integrals.

    a. $\displaystyle \int \cos(x) - \frac{3}{x^5}\,dx$

    b. $\displaystyle \int_{-3}^{4} \cos(x) - \frac{3}{x^5}\,dx$

    c. $\displaystyle \int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx$

a. $\displaystyle \int \cos(x) - \frac{3}{x^5}\,dx$

This is just an indefinite integral and by this point we should be comfortable doing them so here is the answer to this part.

$$\int \cos(x) - \frac{3}{x^5}\,dx = \int \cos(x) - 3x^{-5}\,dx = \sin(x) + \tfrac{3}{4}x^{-4} + c = \boxed{\sin(x) + \frac{3}{4x^4} + c}$$

Don't forget to add on the "+c" since we are doing an indefinite integral!

b. $\displaystyle\int_{-3}^{4} \cos(x) - \frac{3}{x^5}\,dx$

Recall that in order to do a definite integral the integrand (*i.e.* the function we are integrating) must be continuous on the interval over which we are integrating, $\left[-3,4\right]$ in this case.

We can clearly see that the second term will have division by zero at $x = 0$ and $x = 0$ is in the interval over which we are integrating and so this function is not continuous on the interval over which we are integrating.

Therefore, this integral cannot be done.

c. $\displaystyle\int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx$

Now, the function still has a division by zero problem in the second term at $x = 0$. However, unlike the previous part $x = 0$ does not fall in the interval over which we are integrating, $\left[1,4\right]$ in this case.

This integral can therefore be done. Here is the work for this integral.

$$\int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx = \int_{1}^{4} \cos(x) - 3x^{-5}\,dx = \left(\sin(x) + \frac{3}{4x^4}\right)\Bigg|_{1}^{4}$$

$$= \sin(4) + \frac{3}{4(4^4)} - \left(\sin(1) - \frac{3}{4(1^4)}\right)$$

$$= \sin(4) + \frac{3}{1024} - \left(\sin(1) - \frac{3}{4}\right) = \boxed{\sin(4) - \sin(1) - \frac{765}{1024}}$$

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2. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{1}^{6} 12x^3 - 9x^2 + 2\,dx$$

Step 1

First we need to integrate the function.

$$\int_1^6 12x^3 - 9x^2 + 2\,dx = \left(3x^4 - 3x^3 + 2x\right)\Big|_1^6$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_1^6 12x^3 - 9x^2 + 2\,dx = 3252 - 2 = \boxed{3250}$$

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3. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-2}^1 5z^2 - 7z + 3\,dz$$

Step 1
First we need to integrate the function.

$$\int_{-2}^1 5z^2 - 7z + 3\,dz = \left(\tfrac{5}{3}z^3 - \tfrac{7}{2}z^2 + 3z\right)\Big|_{-2}^1$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{-2}^{1} 5z^2 - 7z + 3\,dz = \tfrac{7}{6} - \left(-\tfrac{100}{3}\right) = \boxed{\tfrac{69}{2}}$$

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4. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{3}^{0} 15w^4 - 13w^2 + w\,dw$$

Step 1
First, do not get excited about the fact that the lower limit of integration is a larger number than the upper limit of integration. The problem works in exactly the same way.

So, we need to integrate the function.

$$\int_{3}^{0} 15w^4 - 13w^2 + w\,dw = \left(3w^5 - \tfrac{13}{3}w^2 + \tfrac{1}{2}w^2\right)\Big|_{3}^{0}$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{3}^{0} 15w^4 - 13w^2 + w\,dw = 0 - \tfrac{1233}{2} = \boxed{-\tfrac{1233}{2}}$$

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5. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{1}^{4} \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\,dt$$

Step 1
First we need to integrate the function.

$$\int_1^4 \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\, dt = \int_1^4 8t^{-\frac{1}{2}} - 12t^{\frac{3}{2}}\, dt = \left(16t^{\frac{1}{2}} - \frac{24}{5}t^{\frac{5}{2}}\right)\Big|_1^4$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_1^4 \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\, dt = -\frac{608}{5} - \frac{56}{5} = \boxed{-\frac{664}{5}}$$

6. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_1^2 \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\, dz$$

Step 1
First we need to integrate the function.

$$\int_1^2 \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\, dz = \int_1^2 \frac{1}{7}\frac{1}{z} + \frac{1}{4}z^{\frac{2}{3}} - \frac{1}{2}z^{-3}\, dz = \left(\frac{1}{7}\ln|z| + \frac{3}{20}z^{\frac{5}{3}} + \frac{1}{4}z^{-2}\right)\Big|_1^2$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{1}^{2} \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3} \, dz = \left(\frac{1}{7}\ln(2) + \frac{3}{20}\left(2^{\frac{5}{3}}\right) + \frac{1}{16} \right) - \left(\frac{1}{7}\ln(1) + \frac{2}{5} \right) = \boxed{\frac{1}{7}\ln(2) + \frac{3}{20}\left(2^{\frac{5}{3}}\right) - \frac{27}{80}}$$

Don't forget that $\ln(1) = 0$! Also, don't get excited about "messy" answers like this. They happen on occasion.

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7. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-2}^{4} x^6 - x^4 + \frac{1}{x^2} \, dx$$

Solution

In this case note that the third term will have division by zero at $x = 0$ and this is in the interval we are integrating over, $[-2, 4]$ and hence is not continuous on this interval.

Therefore, this integral cannot be done.

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8. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-4}^{-1} x^2 (3 - 4x) \, dx$$

Step 1
In this case we'll first need to multiply out the integrand before we actually do the integration. Doing that integrating the function gives,

$$\int_{-4}^{-1} x^2 (3 - 4x) \, dx = \int_{-4}^{-1} 3x^2 - 4x^3 \, dx = \left(x^3 - x^4 \right)\Big|_{-4}^{-1}$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{-4}^{-1} x^2 (3-4x)\,dx = -2 - (-320) = \boxed{318}$$

9. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{2}^{1} \frac{2y^3 - 6y^2}{y^2}\,dy$$

Step 1
In this case we'll first need to simplify the integrand to remove the quotient before we actually do the integration. Doing that integrating the function gives,

$$\int_{2}^{1} \frac{2y^3 - 6y^2}{y^2}\,dy = \int_{2}^{1} 2y - 6\,dy = \left(y^2 - 6y \right)\Big|_{2}^{1}$$

Do not get excited about the fact that the lower limit of integration is larger than the upper limit of integration. This will happen on occasion and the integral works in exactly the same manner as we've been doing them.

Also, recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{2}^{1} \frac{2y^{3}-6y^{2}}{y^{2}}\,dy = -5-(-8) = \boxed{3}$$

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10. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{0}^{\frac{\pi}{2}} 7\sin(t)-2\cos(t)\,dt$$

Step 1
First we need to integrate the function.

$$\int_{0}^{\frac{\pi}{2}} 7\sin(t)-2\cos(t)\,dt = \left. \left(-7\cos(t)-2\sin(t)\right)\right|_{0}^{\frac{\pi}{2}}$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{0}^{\frac{\pi}{2}} 7\sin(t)-2\cos(t)\,dt = -2-(-7) = \boxed{5}$$

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11. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{0}^{\pi} \sec(z)\tan(z)-1\,dz$$

Solution

Be careful with this integral. Recall that,

$$\sec(z) = \frac{1}{\cos(z)} \qquad\qquad \tan(z) = \frac{\sin(z)}{\cos(z)}$$

Also recall that $\cos\left(\frac{\pi}{2}\right) = 0$ and that $x = \frac{\pi}{2}$ is in the interval we are integrating over, $[0, \pi]$ and hence is not continuous on this interval.

Therefore, this integral cannot be done.

It is often easy to overlook these kinds of division by zero problems in integrands when the integrand is not explicitly written as a rational expression. So, be careful and don't forget that division by zero can sometimes be "hidden" in the integrand!

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12. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw$$

Step 1
First notice that even though we do have some "hidden" rational expression here (in the definitions of the trig functions) neither cosine nor sine is zero in the interval we are integrating over and so both terms are continuous over the interval.

Therefore all we need to do integrate the function.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw = \left(2\tan(w) + 8\csc(w)\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw = \left(\frac{16}{\sqrt{3}} + 2\sqrt{3}\right) - \left(16 + \frac{2}{\sqrt{3}}\right) = \boxed{\frac{14}{\sqrt{3}} + 2\sqrt{3} - 16}$$

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13. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx$$

Step 1
First we need to integrate the function.

$$\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx = \left(\mathbf{e}^x + \tan^{-1}(x)\right)\Big|_0^2$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx = \left(\mathbf{e}^2 + \tan^{-1}(2)\right) - \left(\mathbf{e}^0 + \tan^{-1}(0)\right) = \boxed{\mathbf{e}^2 + \tan^{-1}(2) - 1}$$

Note that $\tan^{-1}(0) = 0$ but $\tan^{-1}(2)$ doesn't have a "nice" answer and so was left as is.

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14. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy$$

Step 1
First we need to integrate the function.

$$\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy = \left(7\mathbf{e}^y + 2\ln|y|\right)\Big|_{-5}^{-2}$$

Recall that we don't need to add the "+c" in the definite integral case as it will just cancel in the next step.

Step 2
The final step is then just to do the evaluation.

We'll leave the basic arithmetic to you to verify and only show the results of the evaluation. Make sure that you evaluate the upper limit first and then subtract off the evaluation at the lower limit.

Here is the answer for this problem.

$$\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy = \left(7\mathbf{e}^{-2} + 2\ln|-2|\right) - \left(7\mathbf{e}^{-5} + 2\ln|-5|\right) = \boxed{7\left(\mathbf{e}^{-2} - \mathbf{e}^{-5}\right) + 2\left(\ln(2) - \ln(5)\right)}$$

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15. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_0^4 f(t)\,dt \quad \text{where } f(t) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \le 1 \end{cases}$$

Hint : Recall that integrals we can always "break up" an integral as follows,

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

See if you can find a good choice for "*c*" that will make this integral doable.

Step 1

This integral can't be done as a single integral give the obvious change of the function at $t=1$ which is in the interval over which we are integrating. However, recall that we can always break up an integral at any point and $t=1$ seems to be a good point to do this.

Breaking up the integral at $t=1$ gives,

$$\int_0^4 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^4 f(t)\,dt$$

So, in the first integral we have $0 \le t \le 1$ and so we can use $f(t)=1-3t^2$ in the first integral. Likewise, in the second integral we have $1 \le t \le 4$ and so we can use $f(t)=2t$ in the second integral.

Making these function substitutions gives,

$$\int_0^4 f(t)\,dt = \int_0^1 1-3t^2\,dt + \int_1^4 2t\,dt$$

Step 2
All we need to do at this point is evaluate each integral. Here is that work.

$$\int_0^4 f(t)\,dt = \int_0^1 1-3t^2\,dt + \int_1^4 2t\,dt = \left(t-t^3\right)\Big|_0^1 + t^2\Big|_1^4 = [0-0]+[16-1]=\boxed{15}$$

16. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-6}^1 g(z)\,dz \text{ where } g(z)=\begin{cases} 2-z & z>-2 \\ 4\mathbf{e}^z & z\le -2 \end{cases}$$

Hint : Recall that integrals we can always "break up" an integral as follows,

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

See if you can find a good choice for "c" that will make this integral doable.

Step 1

This integral can't be done as a single integral give the obvious change of the function at $z = -2$ which is in the interval over which we are integrating. However, recall that we can always break up an integral at any point and $z = -2$ seems to be a good point to do this.

Breaking up the integral at $z = -2$ gives,

$$\int_{-6}^{1} g(z)\, dz = \int_{-6}^{-2} g(z)\, dz + \int_{-2}^{1} g(z)\, dz$$

So, in the first integral we have $-6 \le z \le -2$ and so we can use $g(z) = 4\mathbf{e}^z$ in the first integral. Likewise, in the second integral we have $-2 \le z \le 1$ and so we can use $g(z) = 2 - z$ in the second integral.

Making these function substitutions gives,

$$\int_{-6}^{1} g(z)\, dz = \int_{-6}^{-2} 4\mathbf{e}^z\, dz + \int_{-2}^{1} 2 - z\, dz$$

Step 2
All we need to do at this point is evaluate each integral. Here is that work.

$$\int_{-6}^{1} g(z)\, dz = \int_{-6}^{-2} 4\mathbf{e}^z\, dz + \int_{-2}^{1} 2 - z\, dz = \left(4\mathbf{e}^z\right)\Big|_{-6}^{-2} + \left(2z - \tfrac{1}{2} z^2\right)\Big|_{-2}^{1}$$
$$= \left[4\mathbf{e}^{-2} - 4\mathbf{e}^{-6}\right] + \left[\tfrac{3}{2} - (-6)\right] = \boxed{4\mathbf{e}^{-2} - 4\mathbf{e}^{-6} + \tfrac{15}{2}}$$

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17. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{3}^{6} |2x - 10|\, dx$$

Hint : In order to do this integral we need to "remove" the absolute value bars from the integrand and we should know how to do that by this point.

Step 1
We'll need to "remove" the absolute value bars in order to do this integral. However, in order to do that we'll need to know where $2x - 10$ is positive and negative.

Since $2x - 10$ is the equation of a line is should be fairly clear that we have the following positive/negative nature of the function.

$$x < 5 \qquad \Rightarrow \qquad 2x - 10 < 0$$
$$x > 5 \qquad \Rightarrow \qquad 2x - 10 > 0$$

Step 2

So, to remove the absolute value bars all we need to do then is break the integral up at $x = 5$.

$$\int_3^6 |2x - 10|\, dx = \int_3^5 |2x - 10|\, dx + \int_5^6 |2x - 10|\, dx$$

So, in the first integral we have $3 \le x \le 5$ and so we have $|2x - 10| = -(2x - 10)$ in the first integral. Likewise, in the second integral we have $5 \le x \le 6$ and so we have $|2x - 10| = 2x - 10$ in the second integral. Or,

$$\int_3^6 |2x - 10|\, dx = \int_3^5 -(2x - 10)\, dx + \int_5^6 2x - 10\, dx$$

Step 3

All we need to do at this point is evaluate each integral. Here is that work.

$$\int_3^6 |2x - 10|\, dx = \int_3^5 -2x + 10\, dx + \int_5^6 2x - 10\, dx = \left(-x^2 + 10x\right)\Big|_3^5 + \left(x^2 - 10x\right)\Big|_5^6$$
$$= [25 - 21] + [-24 - (-25)] = \boxed{5}$$

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18. Evaluate the following integral, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

$$\int_{-1}^0 |4w + 3|\, dw$$

Hint : In order to do this integral we need to "remove" the absolute value bars from the integrand and we should know how to do that by this point.

Step 1

We'll need to "remove" the absolute value bars in order to do this integral. However, in order to do that we'll need to know where $4w + 3$ is positive and negative.

Since $4w + 3$ is the equation a line is should be fairly clear that we have the following positive/negative nature of the function.

$$w < -\tfrac{3}{4} \qquad \Rightarrow \qquad 4w + 3 < 0$$
$$w > -\tfrac{3}{4} \qquad \Rightarrow \qquad 4w + 3 > 0$$

Step 2

So, to remove the absolute value bars all we need to do then is break the integral up at $w = -\tfrac{3}{4}$.

$$\int_{-1}^{0} |4w + 3| \, dw = \int_{-1}^{-\frac{3}{4}} |4w + 3| \, dw + \int_{-\frac{3}{4}}^{0} |4w + 3| \, dw$$

So, in the first integral we have $-1 \le w \le -\tfrac{3}{4}$ and so we have $|4w + 3| = -(4w + 3)$ in the first integral. Likewise, in the second integral we have $-\tfrac{3}{4} \le w \le 0$ and so we have $|4w + 3| = 4w + 3$ in the second integral. Or,

$$\int_{-1}^{0} |4w + 3| \, dw = \int_{-1}^{-\frac{3}{4}} -(4w + 3) \, dw + \int_{-\frac{3}{4}}^{0} 4w + 3 \, dw$$

Step 3

All we need to do at this point is evaluate each integral. Here is that work.

$$\int_{-1}^{0} |4w + 3| \, dw = \int_{-1}^{-\frac{3}{4}} -4w - 3 \, dw + \int_{-\frac{3}{4}}^{0} 4w + 3 \, dw = \left(-2w^2 - 3w \right)\Big|_{-1}^{-\frac{3}{4}} + \left(2w^2 + 3w \right)\Big|_{-\frac{3}{4}}^{0}$$
$$= \left[\tfrac{9}{8} - 1 \right] + \left[0 - \left(-\tfrac{9}{8} \right) \right] = \boxed{\tfrac{5}{4}}$$