

Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Applications of Integrals

Average Function Value

1. Determine f_{avg} for $f(x) = 8x - 3 + 5e^{2-x}$ on $[0, 2]$.

Solution

There really isn't all that much to this problem other than use the formula given in the notes for this section.

$$f_{\text{avg}} = \frac{1}{2-0} \int_0^2 8x - 3 + 5e^{2-x} dx = \frac{1}{2} \left(4x^2 - 3x - 5e^{2-x} \right) \Big|_0^2 = \boxed{\frac{1}{2}(5 + 5e^2)}$$

Note that we are assuming your integration skills are pretty good at this point and won't be showing many details of the actual integration process. This includes not showing substitutions such as the substitution needed for the third term (you did catch that correct?).

2. Determine f_{avg} for $f(x) = \cos(2x) - \sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2}, \pi\right]$.

Solution

There really isn't all that much to this problem other than use the formula given in the notes for this section.

$$f_{\text{avg}} = \frac{1}{\pi - \left(-\frac{\pi}{2}\right)} \int_{-\frac{\pi}{2}}^{\pi} \cos(2x) - \sin\left(\frac{x}{2}\right) dx = \frac{2}{3\pi} \left(\frac{1}{2} \sin(2x) + 2 \cos\left(\frac{x}{2}\right) \right) \Big|_{-\frac{\pi}{2}}^{\pi} = \boxed{-\frac{2\sqrt{2}}{3\pi}}$$

Note that we are assuming your integration skills are pretty good at this point and won't be showing many details of the actual integration process. This includes not showing either of the substitutions needed for the integral (you did catch both of them correct?).

3. Find f_{avg} for $f(x) = 4x^2 - x + 5$ on $[-2, 3]$ and determine the value(s) of c in $[-2, 3]$ for which $f(c) = f_{\text{avg}}$.

Step 1

First we need to use the formula for the notes in this section to find f_{avg} .

$$f_{\text{avg}} = \frac{1}{3 - (-2)} \int_{-2}^3 4x^2 - x + 5 dx = \frac{1}{5} \left(\frac{4}{3} x^3 - \frac{1}{2} x^2 + 5x \right) \Big|_{-2}^3 = \boxed{\frac{83}{6}}$$

Step 2

Note that for the second part of this problem we are really just asking to find the value of c that satisfies the Mean Value Theorem for Integrals.

There really isn't much to do here other than solve $f(c) = f_{\text{avg}}$.

$$4c^2 - c + 5 = \frac{83}{6}$$

$$4c^2 - c - \frac{53}{6} = 0 \quad \Rightarrow \quad c = \frac{1 \pm \sqrt{1 - 4(4)\left(-\frac{53}{6}\right)}}{2(4)} = \frac{1 \pm \sqrt{\frac{427}{3}}}{2(4)} = \boxed{-1.3663, 1.6163}$$

So, unlike the example from the notes both of the numbers that we found here are in the interval and so are both included in the answer.

