Section 6-2 : Area Between Curves

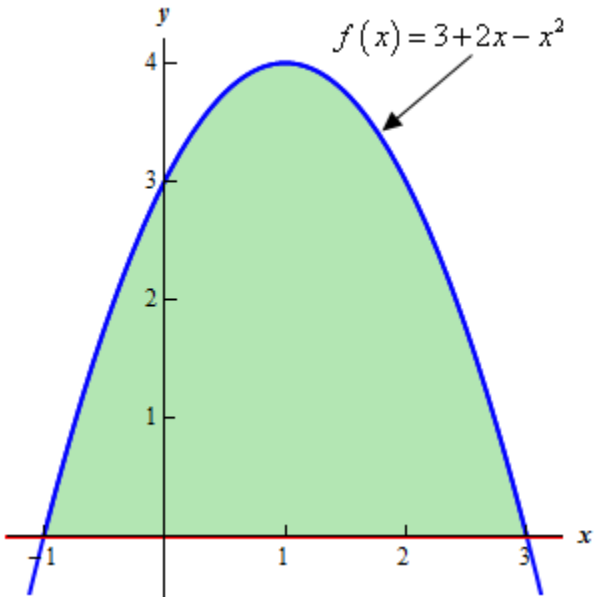
1. Determine the area below $f(x)=3+2x-x^2$ and above the *x*-axis.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
It should be clear from the graph that the upper function is the parabola (*i.e.* $y=3+2x-x^2$) and the lower function is the *x*-axis (*i.e.* $y=0$).

Since we weren't given any limits on *x* in the problem statement we'll need to get those. From the graph it looks like the limits are (probably) $-1\le x\le 3$. However, we should never just assume that our graph is accurate or that we were able to read it accurately. For all we know the limits are close to those we guessed from the graph but are in fact slightly different.

So, to determine if we guessed the limits correctly from the graph let's find them directly. The limits are where the parabola crosses the *x*-axis and so all we need to do is set the parabola equal to zero (*i.e.* where it crosses the line $y=0$) and solve. Doing this gives,

$$3+2x-x^2=0 \quad \rightarrow \quad -(x+1)(x-3)=0 \quad \rightarrow \quad x=-1,\ \ x=3$$

So, we did guess correctly, but it never hurts to be sure. That is especially true here where finding them directly takes almost no time.

Step 3
At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A=\int_{-1}^{3} 3+2x-x^2\,dx=\left(3x+x^2-\tfrac{1}{3}x^3\right)\Big|_{-1}^{3}=\boxed{\tfrac{32}{3}}$$

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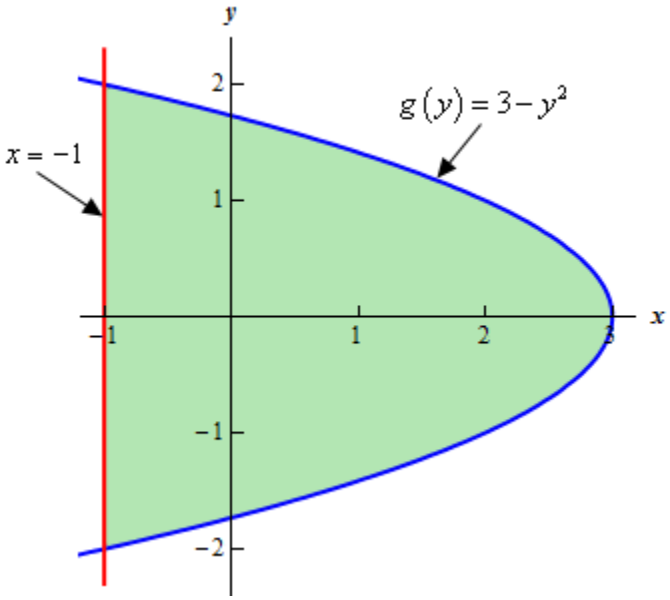
2. Determine the area to the left of $g(y)=3-y^2$ and to the right of $x=-1$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the right/left functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2

It should be clear from the graph that the right function is the parabola (*i.e.* $x=3-y^2$) and the left function is the line $x=-1$.

Since we weren't given any limits on *y* in the problem statement we'll need to get those. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the limits from the graph. This is especially true when the intersection points of the two curves (*i.e.* the limits on *y* that we need) do not occur on an axis (as they don't in this case).

So, to determine the intersection points correctly we'll need to find them directly. The intersection points are where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$3-y^2=-1 \quad \rightarrow \quad y^2=4 \quad \rightarrow \quad y=-2,\;\; y=2$$

So, the limits on *y* are : $-2\le y\le 2$.

Step 3

At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A=\int_{-2}^{2} 3-y^2-(-1)\,dy=\int_{-2}^{2} 4-y^2\,dy=\left(4y-\tfrac{1}{3}y^3\right)\Big|_{-2}^{2}=\boxed{\tfrac{32}{3}}$$

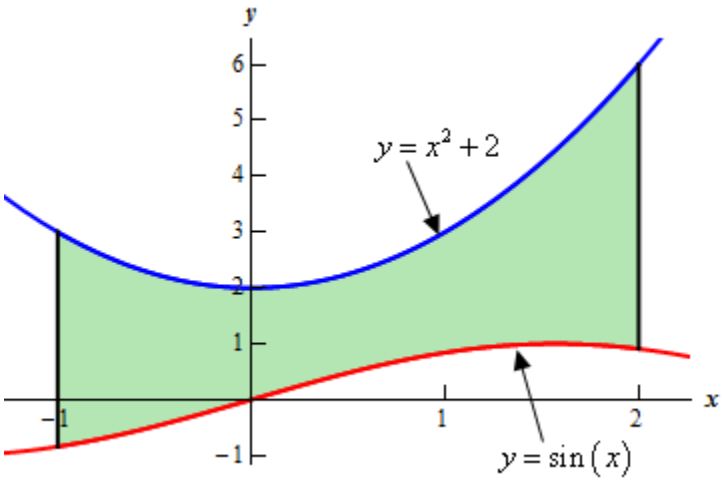
3. Determine the area of the region bounded by $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
It should be clear from the graph that the upper function is $y = x^2 + 2$ and the lower function is $y = \sin(x)$.

Next, we were given limits on *x* in the problem statement and we can see that the two curves do not intersect in that range. Note that this is something that we can't always guarantee and so we need the graph to verify if the curves intersect or not. We should never just assume that because limits on *x* were given in the problem statement that the curves will not intersect anywhere between the given limits.

So, because the curves do not intersect we will be able to find the area with a single integral using the limits : $-1 \le x \le 2$.

Step 3
At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A=\int_{-1}^{2} x^2+2-\sin(x)\,dx=\left(\tfrac{1}{3}x^3+2x+\cos(x)\right)\Big|_{-1}^{2}=\boxed{9+\cos(2)-\cos(1)=8.04355}$$

Don't forget to set your calculator to radians if you take the answer to a decimal.

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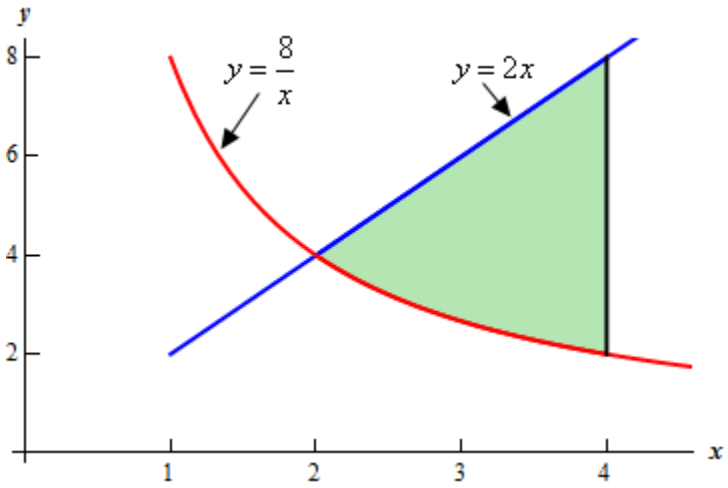
4. Determine the area of the region bounded by $y=\dfrac{8}{x}$, $y=2x$ and $x=4$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



For this problem we were only given one limit on *x* (*i.e.* $x=4$). To determine just what the region we are after recall that we are after a *bounded* region. This means that one of the given curves must be on each boundary of the region.

Therefore, we can't use any portion of the region to the right of the line $x = 4$ because there will never be a boundary on the right of that region.

We also can't take any portion of the region to the left of the intersection point. Because the first function is not continuous at $x = 0$ we can't use any region that includes $x = 0$. Therefore, any portion of the region to the left of the intersection point would have to stop prior to the *y*-axis and any region like that would not have any of the given curves on the left boundary.

The region is then the one shown in graph above. We will take the region to the left of the line $x = 4$ and to the right of the intersection point.

Step 2
We now need to determine the intersection point. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. This is especially true when the intersection point of the two curves does not occur on an axis (as they don't in this case).

So, to determine the intersection point correctly we'll need to find it directly. The intersection point is where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$\frac{8}{x} = 2x \quad \rightarrow \quad x^2 = 4 \quad \rightarrow \quad x = -2, \;\; x = 2$$

Note that while we got two answers here the negative value does not make any sense because to get to that value we would have to go through $x = 0$ and as we discussed above the bounded region cannot contain $x = 0$.

Therefore the limits on *x* are : $2 \le x \le 4$.

It should also be clear from the graph and the limits above that the upper function is $y = 2x$ and the lower function is $y = \frac{8}{x}$.

Step 3
At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A = \int_2^4 2x - \frac{8}{x}\,dx = \left(x^2 - 8\ln|x| \right)\Big|_2^4 = \boxed{12 - 8\ln(4) + 8\ln(2) = 6.4548}$$

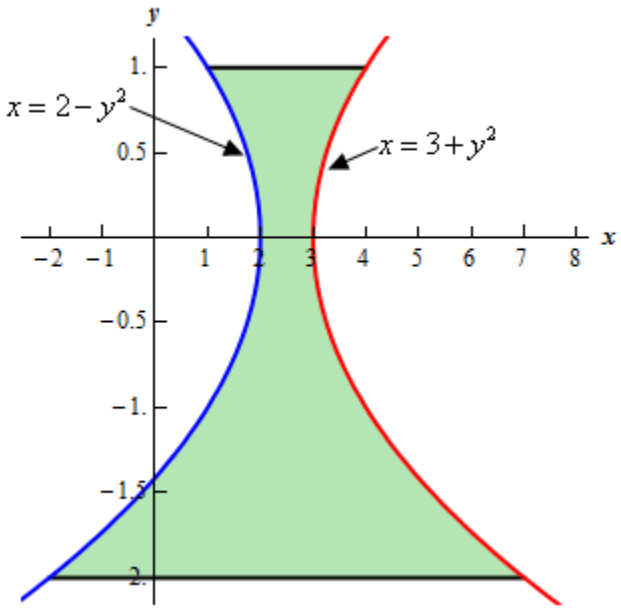
5. Determine the area of the region bounded by $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the right/left functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
It should be clear from the graph that the right function is $x = 3 + y^2$ and the left function is $x = 2 - y^2$.

Next, we were given limits on *y* in the problem statement and we can see that the two curves do not intersect in that range. Note that this is something that we can't always guarantee and so we need the graph to verify if the curves intersect or not. We should never just assume that because limits on *y* were given in the problem statement that the curves will not intersect anywhere between the given limits.

So, because the curves do not intersect we will be able to find the area with a single integral using the limits : $-2 \le y \le 1$.

Step 3
At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A=\int_{-2}^{1} 3+y^2-\left(2-y^2\right)dy=\int_{-2}^{1} 1+2y^2\,dy=\left(y+\tfrac{2}{3}y^3\right)\Big|_{-2}^{1}=\boxed{9}$$

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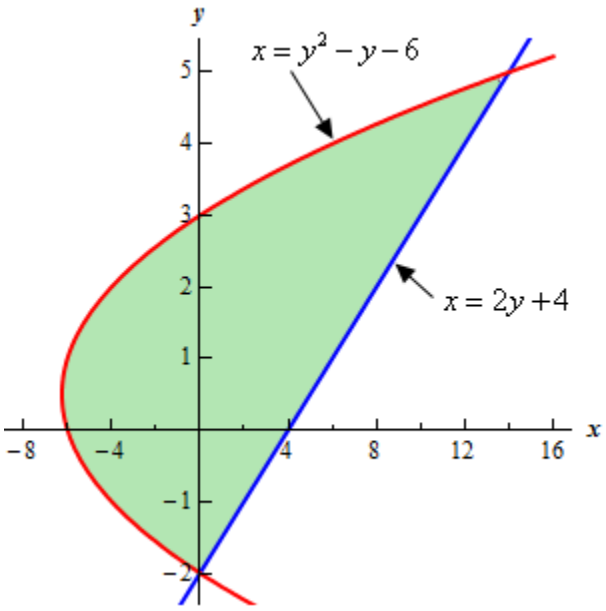
6. Determine the area of the region bounded by $x=y^2-y-6$ and $x=2y+4$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the right/left functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Note that we won't include any portion of the region above the top intersection point or below the bottom intersection point. The region needs to be bounded by one of the given curves on each

boundary. If we went past the top intersection point we would not have an upper bound on the region. Likewise, if we went past the bottom intersection point we would not have a lower bound on the region.

Step 2
It should be clear from the graph that the right function is $x = 2y + 4$ and the left function is $x = y^2 - y - 6$.

Since we weren't given any limits on *y* in the problem statement we'll need to get those. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. This is especially true when the intersection points of the two curves (*i.e.* the limits on *y* that we need) do not occur on an axis (as they don't in this case).

So, to determine the intersection points correctly we'll need to find them directly. The intersection points are where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$y^2 - y - 6 = 2y + 4 \quad \rightarrow \quad y^2 - 3y - 10 = (y - 5)(y + 2) = 0 \quad \rightarrow \quad y = -2, \;\; y = 5$$

Therefore the limits on *y* are : $-2 \le y \le 5$.

Note that you may well have found the intersection points in the first step to help with the graph if you were graphing by hand which is not a bad idea with faced with graphing this kind of region.

Step 3
At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A = \int_{-2}^{5} 2y + 4 - \left(y^2 - y - 6\right) dy = \int_{-2}^{5} 10 + 3y - y^2 \, dy = \left(10y + \tfrac{3}{2}y^2 - \tfrac{1}{3}y^3\right)\Big|_{-2}^{5} = \boxed{\tfrac{343}{6}}$$

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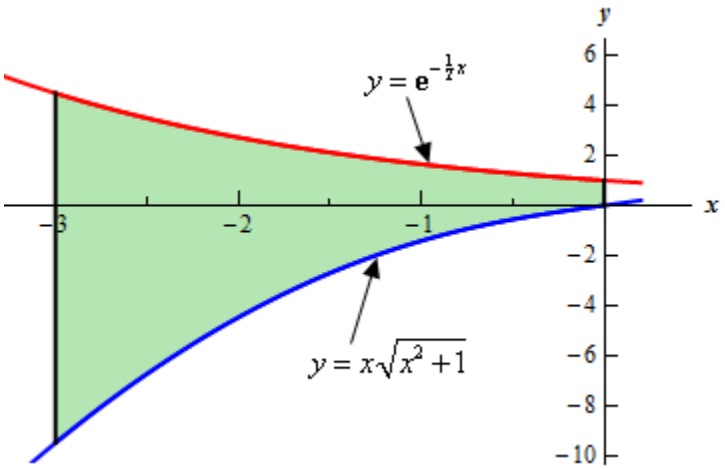
7. Determine the area of the region bounded by $y = x\sqrt{x^2 + 1}$, $y = \mathbf{e}^{-\frac{1}{2}x}$, $x = -3$ and the *y*-axis.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

Note that using a graphing calculator or computer may be needed to deal with the first equation, however you should be able to sketch the graph of the second equation by hand.

Here is a sketch of the bounded region we want to find the area of.



Step 2

It should be clear from the graph that the upper function is $y = \mathbf{e}^{-\frac{1}{2}x}$ and the lower function is $y = x\sqrt{x^2+1}$.

Next, we were given limits on *x* in the problem statement (recall that the *y*-axis is just the line $x = 0$!) and we can see that the two curves do not intersect in that range. Note that this is something that we can't always guarantee and so we need the graph to verify if the curves intersect or not. We should never just assume that because limits on *x* were given in the problem statement that the curves will not intersect anywhere between the given limits.

So, because the curves do not intersect we will be able to find the area with a single integral using the limits : $-3 \le x \le 0$.

Step 3

At this point there isn't much to do other than step up the integral and evaluate it.

We are assuming that you are comfortable with basic integration techniques, including substitution since that will be needed here, so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A = \int_{-3}^{0} \mathbf{e}^{-\frac{1}{2}x} - x\sqrt{x^2+1}\,dx = \left(-2\mathbf{e}^{-\frac{1}{2}x} - \tfrac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}\right)\Big|_{-3}^{0} = \boxed{-\tfrac{7}{3} + 2\mathbf{e}^{\frac{3}{2}} + \tfrac{1}{3}10^{\frac{3}{2}} = 17.17097}$$

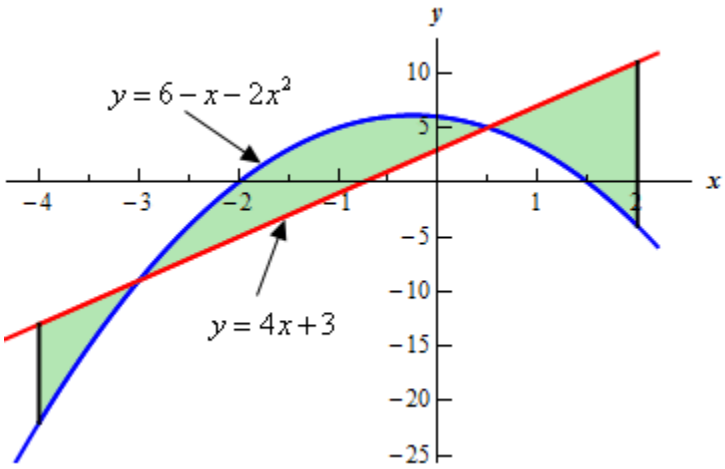
8. Determine the area of the region bounded by $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
In the problem statement we were given two limits on *x*. However, as seen in the sketch of the graph above the curves intersect in this region and the upper/lower functions differ depending on what range of *x*'s we are looking for.

Therefore we'll need to find the intersection points. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. This is especially true when the intersection points of the two curves do not occur on an axis (as they don't in this case).

So, to determine the intersection points correctly we'll need to find them directly. The intersection points are where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$6 - x - 2x^2 = 4x + 3 \quad \rightarrow \quad 2x^2 + 5x - 3 = (2x - 1)(x + 3) = 0 \quad \rightarrow \quad x = -3, \;\; x = \tfrac{1}{2}$$

Note that you may well have found the intersection points in the first step to help with the graph if you were graphing by hand which is not a bad idea with faced with graphing this kind of region.

So, from the graph then it looks like we'll need three integrals since there are three ranges of x ($-4 \le x \le -3$, $-3 \le x \le \frac{1}{2}$ and $\frac{1}{2} \le x \le 2$) for which the upper/lower functions are different.

Step 3
At this point there isn't much to do other than step up the integrals and evaluate them.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$\begin{aligned} A &= \int_{-4}^{-3} 4x+3-\left(6-x-2x^2\right)dx + \int_{-3}^{\frac{1}{2}} 6-x-2x^2-\left(4x+3\right)dx + \int_{\frac{1}{2}}^{2} 4x+3-\left(6-x-2x^2\right)dx \\ &= \int_{-4}^{-3} 2x^2+5x-3\,dx + \int_{-3}^{\frac{1}{2}} 3-5x-2x^2\,dx + \int_{\frac{1}{2}}^{2} 2x^2+5x-3\,dx \\ &= \left(\tfrac{2}{3}x^3+\tfrac{5}{2}x^2-3x\right)\Big|_{-4}^{-3} + \left(3x-\tfrac{5}{2}x^2-\tfrac{2}{3}x^3\right)\Big|_{-3}^{\frac{1}{2}} + \left(\tfrac{2}{3}x^3+\tfrac{5}{2}x^2-3x\right)\Big|_{\frac{1}{2}}^{2} \\ &= \tfrac{25}{6}+\tfrac{343}{24}+\tfrac{81}{8} = \boxed{\tfrac{343}{12}} \end{aligned}$$

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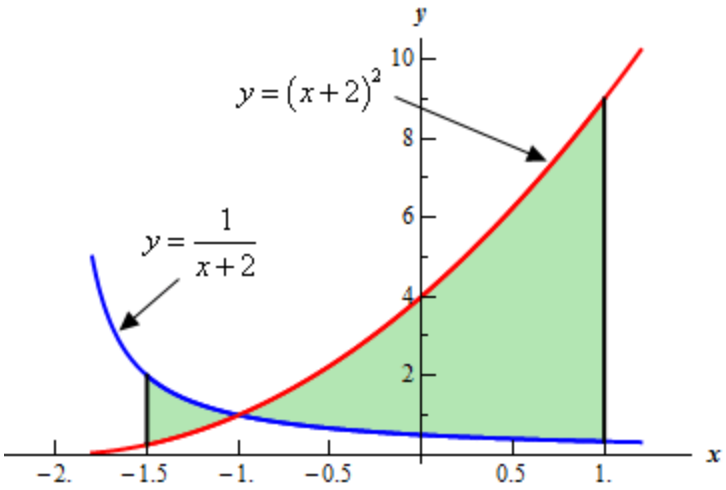
9. Determine the area of the region bounded by $y = \dfrac{1}{x+2}$, $y = \left(x+2\right)^2$, $x = -\dfrac{3}{2}$, $x = 1$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the upper/lower functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
In the problem statement we were given two limits on *x*. However, as seen in the sketch of the graph above the curves intersect in this region and the upper/lower functions differ depending on what range of *x*'s we are looking for.

Therefore, we'll need to find the intersection point. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. This is especially true when the intersection point of the two curves does not occur on an axis (as they don't in this case).

So, to determine the intersection points correctly we'll need to find it directly. The intersection point is where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$\frac{1}{x+2}=(x+2)^2 \quad \rightarrow \quad (x+2)^3=1 \quad \rightarrow \quad x+2=\sqrt[3]{1}=1 \quad \rightarrow \quad x=-1$$

So, from the graph then it looks like we'll need two integrals since there are two ranges of *x* ($-\frac{3}{2}\le x\le -1$ and $-1\le x\le 1$) for which the upper/lower functions are different.

Step 3
At this point there isn't much to do other than step up the integrals and evaluate them.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$A=\int_{-\frac{3}{2}}^{-1}\frac{1}{x+2}-(x+2)^2\,dx+\int_{-1}^{1}(x+2)^2-\frac{1}{x+2}\,dx$$
$$=\left(\ln|x+2|-\tfrac{1}{3}(x+2)^3\right)\Big|_{-\frac{3}{2}}^{-1}+\left(\tfrac{1}{3}(x+2)^3-\ln|x+2|\right)\Big|_{-1}^{1}$$
$$=\left[-\tfrac{7}{24}-\ln\left(\tfrac{1}{2}\right)\right]+\left[\tfrac{26}{3}-\ln(3)\right]=\boxed{\tfrac{67}{8}-\ln\left(\tfrac{1}{2}\right)-\ln(3)=7.9695}$$

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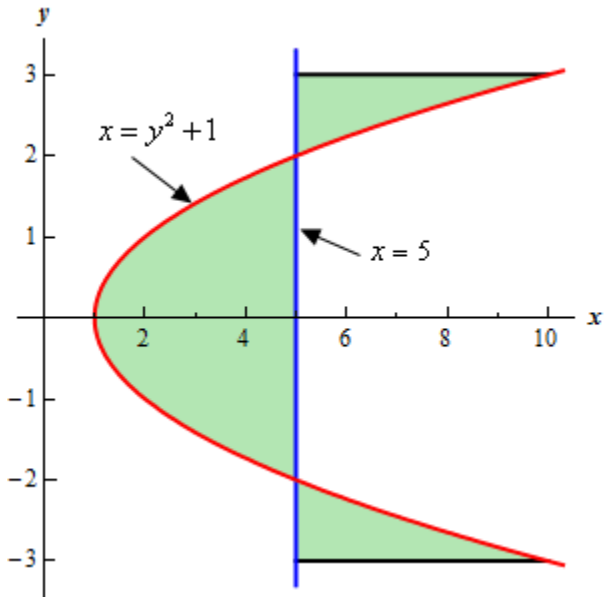
10. Determine the area of the region bounded by $x=y^2+1$, $x=5$, $y=-3$ and $y=3$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the right/left functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
In the problem statement we were given two limits on *y*. However, as seen in the sketch of the graph above the curves intersect in this region and the right/left functions differ depending on what range of *y*'s we are looking for.

Therefore, we'll need to find the intersection points. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. This is especially true when the intersection points of the two curves do not occur on an axis (as they don't in this case).

So, to determine the intersection points correctly we'll need to find them directly. The intersection points are where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$y^2+1=5 \quad \rightarrow \quad y^2=4 \quad \rightarrow \quad y=-2,\ \ y=2$$

Note that you may well have found the intersection points in the first step to help with the graph if you were graphing by hand which is not a bad idea with faced with graphing this kind of region.

So, from the graph then it looks like we'll need three integrals since there are three ranges of *x* ($-3\le x\le -2$, $-2\le x\le 2$ and $2\le x\le 3$) for which the right/left functions are different.

Step 3
At this point there isn't much to do other than step up the integrals and evaluate them.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$\begin{aligned} A &= \int_{-3}^{-2} y^2+1-5\,dy + \int_{-2}^{2} 5-\left(y^2+1\right)dy + \int_{2}^{3} y^2+1-5\,dy \\ &= \int_{-3}^{-2} y^2-4\,dy + \int_{-2}^{2} 4-y^2\,dy + \int_{2}^{3} y^2-4\,dy \\ &= \left(\tfrac{1}{3}y^3-4y\right)\Big|_{-3}^{-2} + \left(4y-\tfrac{1}{3}y^3\right)\Big|_{-2}^{-2} + \left(\tfrac{1}{3}y^3-4y\right)\Big|_{2}^{3} \\ &= \tfrac{7}{3}+\tfrac{32}{3}+\tfrac{7}{3} = \boxed{\tfrac{46}{3}} \end{aligned}$$

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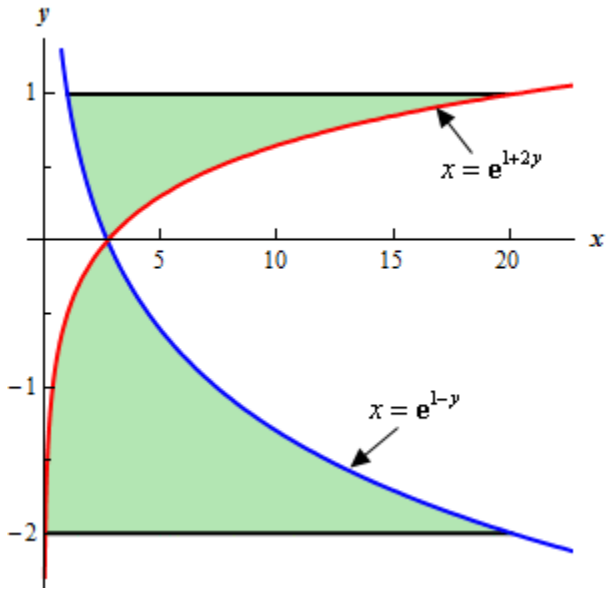
11. Determine the area of the region bounded by $x=\mathbf{e}^{1+2y}$, $x=\mathbf{e}^{1-y}$, $y=-2$ and $y=1$.

Hint : It's generally best to sketch the bounded region that we want to find the area of before starting the actual problem. Having the sketch of the graph will usually help with determining the right/left functions and the limits for the integral.

Step 1
Let's start off with getting a sketch of the region we want to find the area of.

We are assuming that, at this point, you are capable of graphing most of the basic functions that we're dealing with in these problems and so we won't be showing any of the graphing work here.

Here is a sketch of the bounded region we want to find the area of.



Step 2
In the problem statement we were given two limits on *y*. However, as seen in the sketch of the graph above the curves intersect in this region and the right/left functions differ depending on what range of *y*'s we are looking for.

Therefore, we'll need to find the intersection point. However, we should never just assume that our graph is accurate or that we will be able to read it accurately enough to guess the coordinates from the graph. In this case it seems pretty clear from the graph that the intersection point lies on the *x*-axis (and so we can guess the point we need is $y = 0$). However, for all we know the actual intersection point is slightly above or slightly below the *x*-axis and the scale of the graph just makes this hard to see.

So, to determine the intersection points correctly we'll need to find it directly. The intersection point is where the two curves intersect and so all we need to do is set the two equations equal and solve. Doing this gives,

$$\mathbf{e}^{1+2y} = \mathbf{e}^{1-y} \quad \rightarrow \quad \frac{\mathbf{e}^{1+2y}}{\mathbf{e}^{1-y}} = 1 \quad \rightarrow \quad \mathbf{e}^{3y} = 1 \quad \rightarrow \quad y = 0$$

So, from the graph then it looks like we'll need two integrals since there are two ranges of *x* ($-2 \le x \le 0$ and $0 \le x \le 1$) for which the right/left functions are different.

Step 3
At this point there isn't much to do other than step up the integrals and evaluate them.

We are assuming that you are comfortable with basic integration techniques so we'll not be including any discussion of the actual integration process here and we will be skipping some of the intermediate steps.

The area is,

$$\begin{aligned} A &= \int_{-2}^{0} \mathbf{e}^{1-y} - \mathbf{e}^{1+2y}\, dy + \int_{0}^{1} \mathbf{e}^{1+2y} - \mathbf{e}^{1-y}\, dy \\ &= \left(-\mathbf{e}^{1-y} - \tfrac{1}{2}\mathbf{e}^{1+2y} \right)\Big|_{-2}^{0} + \left(\tfrac{1}{2}\mathbf{e}^{1+2y} + \mathbf{e}^{1-y} \right)\Big|_{0}^{1} \\ &= \left[\mathbf{e}^{3} + \tfrac{1}{2}\mathbf{e}^{-3} - \tfrac{3}{2}\mathbf{e} \right] + \left[1 + \tfrac{1}{2}\mathbf{e}^{3} - \tfrac{3}{2}\mathbf{e} \right] = \boxed{22.9983} \end{aligned}$$
