Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Applications of Derivatives

Finding Absolute Extrema

1. Determine the absolute extrema of $f\left(x\right)=8x^3+81x^2-42x-8$ on $\left[-8,2\right]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$f'(x) = 24x^2 + 162x - 42 = 6(4x-1)(x+7) = 0 \qquad \Rightarrow \qquad x = -7, \; x = \tfrac{1}{4}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$x = -7, \quad x = \tfrac{1}{4}$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$f(-8) = 1416 \qquad f(-7) = 1511 \qquad f\left(\tfrac{1}{4}\right) = -13.3125 \qquad f(2) = 296$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 1511 \text{ at } x = -7 \\ \text{Absolute Minimum : } -13.3125 \text{ at } x = \tfrac{1}{4}\end{array}}$$

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2. Determine the absolute extrema of $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-4, 2]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$f'(x) = 24x^2 + 162x - 42 = 6(4x-1)(x+7) = 0 \qquad \Rightarrow \qquad x = -7,\ x = \tfrac{1}{4}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the only critical point that we need is,

$$x = \tfrac{1}{4}$$

Step 3
The next step is to evaluate the function at the critical point from the second step and at the end points of the given interval. Here are those function evaluations.

$$f(-4) = 944 \qquad f\left(\tfrac{1}{4}\right) = -13.3125 \qquad f(2) = 296$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 944 \text{ at } x = -4 \\ \text{Absolute Minimum : } -13.3125 \text{ at } x = \tfrac{1}{4}\end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $x = -7$ we would have gotten the wrong answer for the absolute maximum (check out the previous problem to see this….).

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3. Determine the absolute extrema of $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[-4.5, 4]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$R'(t) = 240t^2 + 20t^3 - 10t^4 = -10t^2(t-6)(t+4) = 0 \qquad \Rightarrow \qquad t = -4, \quad t = 0, \quad t = 6$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$t = -4, \qquad t = 0$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$R(-4.5) = -1548.13 \qquad R(-4) = -1791 \qquad R(0) = 1 \qquad R(4) = 4353$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 4353 \text{ at } t = 4 \\ \text{Absolute Minimum : } -1791 \text{ at } t = -4\end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $t = 6$ we would have gotten the wrong answer for the absolute maximum. Also note that if we'd neglected to check the endpoints at all we also would have gotten the wrong absolute maximum.

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4. Determine the absolute extrema of $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[0,7]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$R'(t) = 240t^2 + 20t^3 - 10t^4 = -10t^2(t-6)(t+4) = 0 \qquad \Rightarrow \qquad t = -4, \quad t = 0, \quad t = 6$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$t = 0, \qquad t = 6$$

Do not get excited about the fact that one of the critical points also happens to be one of the end points of the interval. This happens on occasion.

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$R(0) = 1 \qquad R(6) = 8209 \qquad R(7) = 5832$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 8209 \text{ at } t=6\\ \text{Absolute Minimum : } 1 \text{ at } t=0\end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $t=-4$ we would have gotten the wrong answer for the absolute minimum.

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5. Determine the absolute extrema of $h(z)=4z^3-3z^2+9z+12$ on $[-2,1]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$h'(z)=12z^2-6z+9=0 \qquad \Rightarrow \qquad z=\frac{6\pm\sqrt{-396}}{24}=\frac{1\pm\sqrt{11}\,i}{4}$$

Now, recall that we only work with real numbers here and so we ignore complex roots. Therefore this function has no critical points.

Step 2
Technically the next step is to determine all the critical points that are in the given interval. However, there are no critical points for this function and so there are also no critical points in the given interval.

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. However, since there are no critical points for this function all we need to do is evaluate the function at the end points of the interval.

Here are those function evaluations.

$$h(-2) = -50 \qquad h(1) = 22$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem. That is especially true for this problem as there would be no points to evaluate at without the end points.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 22 \text{ at } z = 1 \\ \text{Absolute Minimum : } -50 \text{ at } z = -2\end{array}}$$

Note that if we hadn't remembered to evaluate the function at the end points of the interval we would not have had an answer for this problem!

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6. Determine the absolute extrema of $g(x) = 3x^4 - 26x^3 + 60x^2 - 11$ on $[1,5]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$g'(x) = 12x^3 - 78x^2 + 120x = 6x(x-4)(2x-5) = 0 \qquad \Rightarrow \qquad x = 0, \quad x = \tfrac{5}{2}, \quad x = 4$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$x = \tfrac{5}{2}, \quad x = 4$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$g(1) = 26 \qquad g\left(\tfrac{5}{2}\right) = 74.9375 \qquad g(4) = 53 \qquad g(5) = 114$$

Do not forget to evaluate the function at the end points! This is one of the biggest mistakes that people tend to make with this type of problem.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 114 \text{ at } x = 5 \\ \text{Absolute Minimum : } 26 \text{ at } x = 1\end{array}}$$

Note that if we hadn't remembered to evaluate the function at the end points of the interval we would have gotten both of the answers incorrect!

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7. Determine the absolute extrema of $Q(x) = (2-8x)^4 (x^2-9)^3$ on $[-3,3]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$\begin{aligned} Q'(x) &= 4(-8)(2-8x)^3(x^2-9)^3 + 3(2x)(2-8x)^4(x^2-9)^2 \\ &= -4(2-8x)^3(x^2-9)^2(20x^2-3x-72) \\ &= 0 \qquad\qquad \Rightarrow \qquad x=\tfrac{1}{4},\quad x=\pm 3,\quad x=\tfrac{3\pm\sqrt{5796}}{40}=-1.8239,\ 1.9739 \end{aligned}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, we need all the critical points from the first step.

$$x=\tfrac{1}{4},\quad x=\pm 3,\quad x=\tfrac{3\pm\sqrt{5796}}{40}=-1.8239,\ 1.9739$$

Do not get excited about the fact that both end points of the interval are also critical points. It happens sometimes and in this case it will reduce the number of computations required in the next step by 2 and that's not a bad thing.

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$Q(-3)=0 \quad Q(-1.8239)=-1.38\times 10^7 \quad Q\left(\tfrac{1}{4}\right)=0 \quad Q(1.9739)=-4.81\times 10^6 \quad Q(3)=0$$

Do not get excited about the large numbers for the two non-zero function values. This is something that is going to happen on occasion and we shouldn't worry about it when it does happen.

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l} \text{Absolute Maximum : 0 at } x=-3,\ x=\tfrac{1}{4},\ x=3 \\ \text{Absolute Minimum : } -1.38\times 10^7 \text{ at } x=-1.8239 \end{array}}$$

Recall that while we can only have one largest possible value (*i.e.* only one absolute maximum) it is completely possible for it to occur at more than one point (3 points in this case).

8. Determine the absolute extrema of $h(w) = 2w^3(w+2)^5$ on $\left[-\frac{5}{2}, \frac{1}{2}\right]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a polynomial and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$\begin{aligned} h'(w) &= 6w^2(w+2)^5 + 10w^3(w+2)^4 \\ &= 4w^2(w+2)^4(4w+3) = 0 \qquad\qquad \Rightarrow \qquad w = 0,\ w = -\tfrac{3}{4},\ w = -2 \end{aligned}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, we need all the critical points from the first step.

$$w = 0,\ w = -\tfrac{3}{4},\ w = -2$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$h\left(-\tfrac{5}{2}\right) = 0.9766 \qquad h(-2) = 0 \qquad h\left(-\tfrac{3}{4}\right) = -2.5749 \qquad h(0) = 0 \qquad h\left(\tfrac{1}{2}\right) = 24.4141$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l} \text{Absolute Maximum : } 24.4141 \text{ at } w = \frac{1}{2} \\ \text{Absolute Minimum : } -2.5749 \text{ at } w = -\frac{3}{4} \end{array}}$$

9. Determine the absolute extrema of $f\left(z\right)=\dfrac{z+4}{2z^2+z+8}$ on $\left[-10,0\right]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a rational expression in which both the numerator and denominator are continuous everywhere. Also notice that the rational expression exists at all points in the interval and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$\begin{aligned} f'\left(z\right) &= \frac{\left(1\right)\left(2z^2+z+8\right)-\left(z+4\right)\left(4z+1\right)}{\left(2z^2+z+8\right)^2} \\ &= \frac{-2\left(z^2+8z-2\right)}{\left(2z^2+z+8\right)^2}=0 \qquad \Rightarrow \qquad z=\tfrac{-8\pm\sqrt{72}}{2}=-4\pm3\sqrt{2}=-8.2426,\ 0.2426 \end{aligned}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the only critical point that we need is,

$$z=-4-3\sqrt{2}=-8.2426$$

Step 3
The next step is to evaluate the function at the critical point from the second step and at the end points of the given interval. Here are those function evaluations.

$$f\left(-10\right)=-\tfrac{1}{33}=-0.0303 \qquad f\left(-8.2426\right)=-0.03128 \qquad f\left(0\right)=\tfrac{1}{2}$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } \frac{1}{2} \text{ at } z = 0 \\ \text{Absolute Minimum : } -0.03128 \text{ at } z = -4 - 3\sqrt{2}\end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $z = -4 + 3\sqrt{2} = 0.2426$ we would have gotten the wrong answer for the absolute maximum.

This problem also shows that we need to be very careful with doing too much rounding of our answers. Had we rounded down to say 2 decimal places we would have been tempted to say that the absolute minimum occurred at two places when in fact one of the points was lower than the other.

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10. Determine the absolute extrema of $A(t) = t^2 (10 - t)^{\frac{2}{3}}$ on $[2, 10.5]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a product of a polynomial and a cube root function. Bother are continuous everywhere and so the product will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$\begin{aligned} A'(t) &= 2t(10-t)^{\frac{2}{3}} + t^2\left(\tfrac{2}{3}\right)(-1)(10-t)^{-\frac{1}{3}} = 2t(10-t)^{\frac{2}{3}} - \frac{2t^2}{3(10-t)^{\frac{1}{3}}} \\ &= \frac{6t(10-t)-2t^2}{3(10-t)^{\frac{1}{3}}} = \frac{60t-8t^2}{3(10-t)^{\frac{1}{3}}} = \frac{4t(15-2t)}{3(10-t)^{\frac{1}{3}}} \\ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad t=0, \quad t=\tfrac{15}{2}, \quad t=10 \end{aligned}$$

Don't forget about critical points where the derivative doesn't exist!

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$t=\tfrac{15}{2}, \quad t=10$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$A(2)=16 \qquad A\left(\tfrac{15}{2}\right)=103.613 \qquad A(10)=0 \qquad A(10.5)=69.4531$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 103.613 \text{ at } t=\tfrac{15}{2} \\ \text{Absolute Minimum : } 0 \text{ at } t=10\end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $t=0$ we would have had the absolute minimum showing up at two places instead of only the one place inside the given interval.

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11. Determine the absolute extrema of $f(y)=\sin\left(\frac{y}{3}\right)+\frac{2y}{9}$ on $[-10,15]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1

First, notice that we are working with the sine function and this is continuous everywhere and so will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$f'(y) = \tfrac{1}{3}\cos\left(\tfrac{y}{3}\right) + \tfrac{2}{9} = 0 \quad \rightarrow \quad \cos\left(\tfrac{y}{3}\right) = -\tfrac{2}{3} \quad \rightarrow \quad \tfrac{y}{3} = \cos^{-1}\left(-\tfrac{2}{3}\right) = 2.3005$$

$$\begin{array}{l} \tfrac{y}{3} = 2.3005 + 2\pi n \\ \tfrac{y}{3} = 3.9827 + 2\pi n \end{array} \quad \Rightarrow \quad \begin{array}{l} y = 6.9016 + 6\pi n \\ y = 11.9481 + 6\pi n \end{array} \quad n = 0, \pm 1, \pm 2, \pm, \ldots$$

If you need some review on solving trig equations please go back to the Review chapter and work some of the problems the Solving Trig Equations sections.

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical points that we need are,

$$y = -6.9016, \quad y = 6.9016, \quad y = 11.9481$$

Note that we got these values by plugging in values of n into the solutions above and checking the results against the given interval.

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$f(-10) = -2.0317 \quad f(-6.9016) = -2.2790 \quad f(6.9016) = 2.2790$$
$$f(11.9481) = 1.9098 \quad f(15) = 2.3744$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l} \text{Absolute Maximum : } 2.3744 \text{ at } y = 15 \\ \text{Absolute Minimum : } -2.2790 \text{ at } y = -6.9016 \end{array}}$$

Note the importance of paying attention to the interval with this problem. Without an interval we would have had (literally) an infinite number of critical points to check. Also, without an interval (as a quick graph of the function would show) there would be no absolute extrema for this function.

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12. Determine the absolute extrema of $g\left(w\right)=\mathbf{e}^{w^3-2w^2-7w}$ on $\left[-\frac{1}{2},\frac{5}{2}\right]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with an exponential function with a polynomial in the exponent. The exponent is continuous everywhere and so we can see that the exponential function will also be continuous everywhere. Therefore the function will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$\begin{aligned} g'\left(w\right) &= \left(3w^2-4w-7\right)\mathbf{e}^{w^3-2w^2-7w} \\ &= \left(w+1\right)\left(3w-7\right)\mathbf{e}^{w^3-2w^2-7w}=0 \qquad \Rightarrow \qquad w=-1,\ w=\tfrac{7}{3} \end{aligned}$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the only critical point that we need is,

$$w=\tfrac{7}{3}$$

Step 3
The next step is to evaluate the function at the critical point from the second step and at the end points of the given interval. Here are those function evaluations.

$$g\left(-\tfrac{1}{2}\right) = \mathbf{e}^{\frac{23}{8}} \qquad g\left(\tfrac{7}{3}\right) = \mathbf{e}^{-\frac{392}{27}} \qquad g\left(\tfrac{5}{2}\right) = \mathbf{e}^{-\frac{115}{8}}$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l} \text{Absolute Maximum : } \mathbf{e}^{\frac{23}{8}} \text{ at } w = -\tfrac{1}{2} \\ \text{Absolute Minimum : } \mathbf{e}^{-\frac{392}{27}} \text{ at } w = \tfrac{7}{3} \end{array}}$$

Note the importance of paying attention to the interval with this problem. Had we neglected to exclude $w = -1$ we would have gotten the absolute maximum wrong.

Also note that we need to be careful with rounding with this problem. Both of the exponentials with negative exponents are very small and rounding could cause some real issues here. However, we don't need to actually do any calculator work for this anyway. Recall that the more negative the exponent is the smaller the exponential will be.

So, because $\frac{392}{27} > \frac{115}{8}$ we must have $\mathbf{e}^{-\frac{392}{27}} < \mathbf{e}^{-\frac{115}{8}}$.

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13. Determine the absolute extrema of $R(x) = \ln\left(x^2 + 4x + 14\right)$ on $[-4, 2]$.

Hint : Just recall the process for finding absolute extrema outlined in the notes for this section and you'll be able to do this problem!

Step 1
First, notice that we are working with a logarithm whose argument is a polynomial (which is continuous everywhere) that is always positive in the interval. Because of this we can see that the function will be continuous on the given interval. Recall that this is important because we now know that absolute extrema will in fact exist by the Extreme Value Theorem!

Now that we know that absolute extrema will in fact exist on the given interval we'll need to find the critical points of the function.

Given that the purpose of this section is to find absolute extrema we'll not be putting much work/explanation into the critical point steps. If you need practice finding critical points please go back and work some problems from that section.

Here are the critical points for this function.

$$R'(x) = \frac{2x+4}{x^2+4x+14} \qquad \Rightarrow \qquad x = -2$$

Step 2
Now, recall that we actually are only interested in the critical points that are in the given interval and so, in this case, the critical point that we need is,

$$x = -2$$

Step 3
The next step is to evaluate the function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$R(-4) = 2.6391 \qquad R(-2) = 2.3026 \qquad R(2) = 3.2581$$

Step 4
The final step is to identify the absolute extrema. So, the answers for this problem are then,

$$\boxed{\begin{array}{l}\text{Absolute Maximum : } 3.2581 \text{ at } x = 2 \\ \text{Absolute Minimum : } 2.3026 \text{ at } x = -2\end{array}}$$