Worksheet 21 **Product and Quotient Rules**

Learn: Product Rule: $f(x)=\square\cdot\Delta \quad \Rightarrow \quad f'(x)=\square\cdot d\Delta+\Delta\cdot d\square$

Quotient Rule: $f(x)=\dfrac{\square}{\Delta} \quad \Rightarrow \quad f'(x)=\dfrac{\Delta\cdot d\square-\square\cdot d\Delta}{\Delta^2}$

Find the derivative.

1. $f(x)=\left(x^3+x+1\right)\left(x^4+x^2+1\right)$

2. $y=\left(x^2+1\right)\left(x^3+1\right)$

3. $y=\dfrac{x^2+5x-1}{x^2}$

4. $g(x)=\dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

Find the equation for the line tangent to the curve at the given point.

5. $y=\dfrac{x^3+1}{2x},\ x=1$

6. $f(x)=\dfrac{x^4+2}{x^2},\ x=-1$

7. Find the equation of the line perpendicular to the tangent to the curve $y=x^3-3x+1$ at the point $(2,3)$.

8. Find an equation of the tangent line to the curve $y=\left(x^3-3x+1\right)(x+2)$ when $x=1$.

9. Find the points on the curve $y=2x^3-3x^2-12x+20$ where the tangent is parallel to the x-axis.

Answers:

1) $f'(x)=(x^3+x+1)(4x^3+2x)+(x^4+x^2+1)(3x^2+1)$

$f'(x)=4x^6+2x^4+4x^4+2x^2+4x^3+2x+3x^6+x^4+3x^4+x^2+3x^2+1$

$\boxed{f'(x)=7x^6+10x^4+4x^3+6x^2+2x+1}$

2) $y'=(x^2+1)(3x^2)+(x^3+1)(2x)$

$y'=3x^4+3x^2+2x^4+2x$

$\boxed{y'=5x^4+3x^2+2x}$

3) $y'=\dfrac{x^2(2x+5)-(x^2+5x-1)(2x)}{(x^2)^2}$

$y'=\dfrac{2x^3+5x^2-2x^3-10x^2+2x}{x^4}$

$\boxed{y'=\dfrac{-5x^2+2x}{x^4}=\dfrac{-5x+2}{x^3}}$

4) $g(x)=\dfrac{x^2+3x+2}{x^2-3x+2}$

$g'(x)=\dfrac{(x^2-3x+2)(2x+3)-(x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}$

$g'(x)=\dfrac{2x^3+3x^2-6x^2-9x+4x+6-2x^3+3x^2-6x^2+9x-4x-6}{(x^2-3x+2)^2}$

$\boxed{g'(x)=\dfrac{-6x^2}{(x^2-3x+2)^2}}$

5) $y'=\dfrac{2x(3x^2)-(x^3+1)(2)}{(2x)^2}$

$\boxed{y'(1)=\dfrac{2(3)-(2)(2)}{4}=\dfrac{1}{2}}$

6) $f'(x)=\dfrac{x^2(4x^3)-(x^4+2)(2x)}{(x^2)^2}$

$\boxed{f'(-1)=\dfrac{1(-4)-(3)(-2)}{1}=2}$

7) $y'=3x^2-3$

$y'(2)=9$

slope of perpendicular is $-\dfrac{1}{9}$

$\boxed{y-3=-\dfrac{1}{9}(x-2)}$

8) $y(1)=-3$

$y'=(x^3-3x+1)(1)+(x+2)(3x^2-3)$

$y'(1)=-1+0=-1$

$\boxed{y-(-3)=-1(x-1)}$

9) parallel to x-axis means slope = 0

$y'=6x^2-6x-12=0$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2;\quad x=-1$

$y=0;\quad y=27$

$(2,0)$ and $(-1,27)$