CALC **BENCHMARK PRACTICE**

NAME _____ PER _____ DATE _____

Directions: **The practice available are categorized into 4 categories. Record all work in your PRACTICE NOTEBOOK and WORK QUICKLY!**

1) Derivatives
2) Application of Derivatives (including graphs)
3) Limits
4) EXTENSION

I –	Identify the question
D –	Determine necessary operation / rule
E –	Eliminate extraneous information
A –	Assess (or check) your answer

REFLECTION: BASED ON TODAY'S PRACTICE, HOW WELL DO YOU THINK YOU WILL DO ON TOMORROW'S BENCHMARK ON A SCALE FROM 1 to 5? Identify which skill(s) you feel the most confident in and which you feel weak in still.

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BM PT.1 Derivatives

NAME _____ PER _____ DATE _____

Criteria of Success: 4 out of 6

1. If $y = x^2e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x(x+2e^x)$ (C) $xe^x(x+2)$

(D) $2x+e^x$ (E) $2x+e$

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1

BM PT.1 Derivatives

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11. An equation of the line tangent to the graph of $f(x) = x(1 - 2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x$ ·

(D) $y = 2x - 3$ (E) $y = 7x - 8$

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

BM PT.2 Application of Derivatives

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Criteria of Success: 4 out of 6

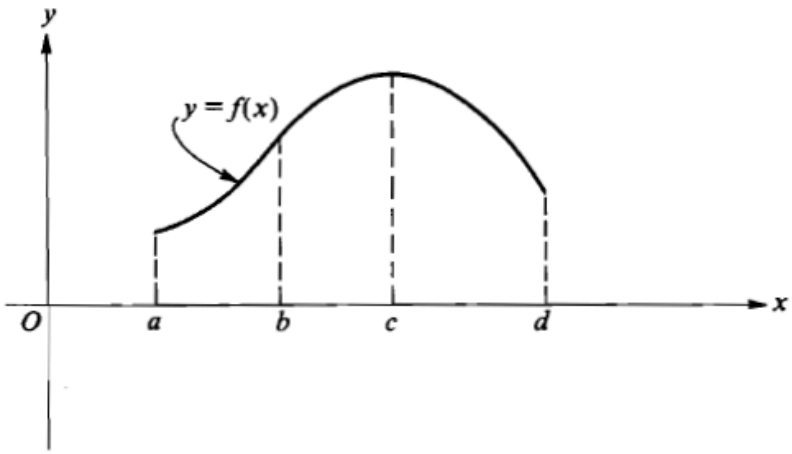
2. What is the domain of the function f given by $f(x) = \dfrac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leqq 2\}$ (C) $\{x: |x| \geqq 2\}$

(D) $\{x: |x| \geqq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geqq 2 \text{ and } x \neq 3\}$

4. The graph of $y = \dfrac{-5}{x - 2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

BM PT.2 Application of Derivatives

NAME _____ PER _____ DATE _____

16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

BM PT.3 LIMITS

NAME _____ PER _____ DATE _____

Criteria of Success: 3 out of 5

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

29. The $\lim_{h \to 0} \frac{\tan 3(x + h) - \tan(3x)}{h}$ is

(A) 0 (B) $3 \sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3 \cot(3x)$ (E) nonexistent

41. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

BM PT.3 LIMITS

NAME _____ **PER** _____ **DATE** _____

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

BM PT.4 EXTENSION (MISC)

NAME _____ PER _____ DATE _____

Criteria of Success: 3 out of 5

9. If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

24. $\frac{d}{dx}(x^{\ln x}) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x - 1})$ (E) $2(\ln x)(x^{\ln x})$

31. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$

37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

BM PT.4 EXTENSION (MISC)

NAME _____ PER _____ DATE _____

Criteria of Success: 2

2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

   (a) Find the acceleration $a(t)$ of the particle at any time t.

   (b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

   (a) Write an equation of the horizontal asymptote for the graph of f.

   (b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

   (c) For what values of x is the graph of f concave down?

   (d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-plane provided below.

ANSWER KEY

Item No.	Correct Answer	Percentage Selecting Each Option[1] (*indicates correct answer) A	B	C	D	E	Percent Omitting Question
1.	C	6%	4%	*88%	1%	1%	1%
2.	D	21	1	1	*60	15	1
3.	A	*83	1	2	8	2	4
4.	E	3	23	1	3	*60	10
5.	A	*89	1	0	2	6	2
6.	D	2	2	4	*83	7	2
7.	D	7	1	7	*79	2	4
8.	B	10	*59	4	9	4	14
9.	E	2	3	7	1	*81	7
10.	C	2	3	*80	3	1	10
11.	A	*61	9	3	6	5	16
12.	B	4	*76	2	15	1	1
13.	A	*70	8	8	4	2	7
14.	D	5	2	7	*67	3	15
15.	B	17	*65	12	2	3	1
16.	C	11	5	*74	1	6	4
17.	D	2	5	8	*72	10	2
18.	E	3	9	6	3	*73	7
19.	B	9	*58	5	1	18	9
20.	C	15	3	*41	4	7	30
21.	C	7	3	*63	3	7	17
22.	C	3	3	*64	5	6	19
23.	B	1	*73	9	4	10	4
24.	C	4	2	*21	42	5	26
25.	B	1	*64	6	20	1	7
26.	E	5	4	7	4	*59	20
27.	E	1	19	11	5	*53	11
28.	C	18	28	*34	4	3	13
29.	B	21	*45	7	1	10	16
30.	A	*57	13	11	3	3	13

ANSWER KEY

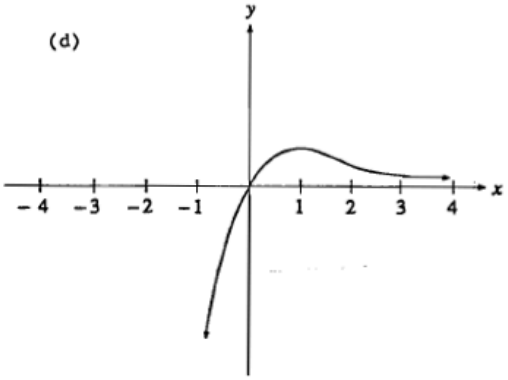
Item No.	Correct Answer	Percentage Selecting Each Option (*indicates correct answer) A	B	C	D	E	Percent Omitting Question
31.	C	7%	20%	*50%	5%	2%	16%
32.	A	*46	11	10	8	9	16
33.	A	*48	8	3	33	5	3
34.	D	15	12	2	*50	14	7
35.	B	6	*25	7	4	18	40
36.	C	1	2	*60	7	9	20
37.	D	6	7	25	*45	4	13
38.	E	1	1	13	6	*47	31
39.	E	37	1	12	2	*34	14
40.	B	9	*50	3	2	4	31
41.	A	*30	9	11	18	21	11
42.	C	5	6	*56	23	6	4
43.	B	11	*24	11	9	7	38
44.	C	19	19	*37	7	7	10
45.	D	18	18	13	*43	8	0

ANSWER KEY

2.

Solution	Distribution of Points
(a) $a(t) = v'(t)$ $= -2\pi \cos(2\pi t)$	(a) 2: for correct differentiation of velocity
(b) $v(t) = 0$ $1 - \sin(2\pi t) = 0$ or $1 = \sin(2\pi t)$ $2\pi t = \frac{\pi}{2} + 2k\pi$, where $k = 0, \pm 1, \pm 2, \ldots$ and $0 \leq t \leq 2$ $\therefore t = \frac{1}{4}$ and $t = \frac{5}{4}$	(b) 3: 1: for $1 - \sin(2\pi t) = 0$ 1: for $t = \frac{1}{4}$ 1: for $t = \frac{5}{4}$

4.

Solution	Distribution of Points
(a) $y = 0$	(a) 1: for correct equation
(b) $f'(x) = 2(-xe^{-x} + e^{-x})$ $= 2e^{-x}(1 - x)$ critical point at $x = 1$ relative maximum at $x = 1$	(b) 3: 1: for correct derivative 1: for critical value for f' 1: for identifying critical point as relative maximum
(c) $f''(x) = 2e^{-x}(-1) + (-2e^{-x})(1-x)$ $= 2e^{-x}(x - 2)$ Concave down when $2e^{-x}(x - 2) < 0$ $(x - 2) < 0$ $x < 2$	(c) 2: 1: for correct $f''(x)$ for $f'(x)$ found in (b) 1: for correct interval
(d) 	(d) 3: for graph consistent with information found in (a), (b), and (c)