AP Calculus BC

CHAPTER 12B WORKSHEET

INFINITE SEQUENCES AND SERIES

Name________________________________

Seat # ________ Date ____________________

Manipulation of power series

Given that

$$h(x) = \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \ldots \text{ on } (-1, 1),$$

answer each of the following questions for problems 1 through 6.

a) Find the first four non-zero terms of the series.
b) Find a formula for the nth term of the series.
c) Find the interval of convergence.

1. $\dfrac{1}{1-x}$

2. $\dfrac{1}{1+x^3}$

3. $\dfrac{-2}{x^2-1}$, given the fact that $\dfrac{-2}{x^2-1} = \dfrac{1}{1+x} + \dfrac{1}{1-x}$

4. $-\dfrac{1}{(x+1)^2}$, given the fact that $-\dfrac{1}{(x+1)^2} = \dfrac{d}{dx}\left(\dfrac{1}{1+x}\right)$

5. $\dfrac{2}{(x+1)^3}$, given the fact that $\dfrac{2}{(x+1)^3} = \dfrac{d^2}{dx^2}\left(\dfrac{1}{1+x}\right)$

6. $\ln(x+1)$, given the fact that $\ln(x+1) = \int \dfrac{1}{1+x} \cdot dx$

7. Use your answer to question #1 and the fact that $\dfrac{x^2}{1-x} = x^2 \cdot \dfrac{1}{1-x}$ to find the first four non-zero terms and a formula for the nth term of the power series for $\dfrac{x^2}{1-x}$

ANSWER KEY

Manipulation of power series

$$h(x)=\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n = 1-x+x^2-x^3+\ldots \text{ on } (-1,\ 1),$$

1. $\dfrac{1}{1-x}$
   a) $1+x+x^2+x^3$
   b) x^n
   c) $(-1,\ 1)$

2. $\dfrac{1}{1+x^3}$
   a) $1-x^3+x^6-x^9$
   b) $(-1)^n x^{3n}$
   c) $(-1,\ 1)$

3. $\dfrac{-2}{x^2-1}$, given the fact that $\dfrac{-2}{x^2-1}=\dfrac{1}{1+x}+\dfrac{1}{1-x}$
   a) $2+2x^2+2x^4+2x^6$
   b) $2x^{2n}$
   c) $(-1,\ 1)$

4. $-\dfrac{1}{(x+1)^2}$, given the fact that $-\dfrac{1}{(x+1)^2}=\dfrac{d}{dx}\left(\dfrac{1}{1+x}\right)$
   a) $-1+2x-3x^2+4x^3$
   b) $(-1)^n nx^{n-1}$ or $(-1)^{n+1}(n+1)x^n$
   c) $(-1,\ 1)$

5. $\dfrac{2}{(x+1)^3}$, given the fact that $\dfrac{2}{(x+1)^3}=\dfrac{d^2}{dx^2}\left(\dfrac{1}{1+x}\right)$
   a) $2-6x+12x^2-20x^3$
   b) $(-1)^n n\cdot(n-1)x^{n-2}$ or $(-1)^n(n+2)\cdot(n+1)x^n$ or $(-1)^{n+1}n\cdot(n+1)x^{n-1}$
   c) $(-1,\ 1)$

6. $\ln(x+1)$, given the fact that $\ln(x+1)=\int\dfrac{1}{1+x}\cdot dx$
   a) $x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}$ (note: $C=0$, since $\ln((0)+1)=0=C+0-0+\ldots$)
   b) $(-1)^n\dfrac{x^{n+1}}{n+1}$ or $(-1)^{n+1}\dfrac{x^n}{n}$
   c) $(-1,\ 1]$

7. $$\frac{x^2}{1-x} = x^2 + x^3 + x^4 + x^5 + \ldots + x^{n+2} + \ldots$$