Name: Answers

AP Calc BC, Unit 6 Test, Practice

Part I:

Match each series with one of the tests for convergence or divergence listed below.
Note: Some types may be used more than once or possibly not at all.
Determine if the series converges or diverges showing that all conditions of the appropriate test have been met.

A. Nth term test (divergence test) B. P-series (or harmonic) C. Alternating D. Geometric

1. $\sum_{n=1}^{\infty} \frac{n^2+1}{n}$ $\lim_{n\to\infty} \frac{n^2+1}{n} = \infty \neq 0$ Type: A Converges or Diverges: _______

2. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^n}$ $\frac{2(2^n)}{5^n} = 2\left(\frac{2}{5}\right)^n$ $r = \frac{2}{5}$ Type: D Converges or Diverges: _______

3. $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ p-series $p > 1$ Type: B Converges or Diverges: _______

4. $\sum_{n=1}^{\infty} \frac{(-1)^n(\sqrt{n})}{n+1}$ $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = 0$ $\frac{\sqrt{n}}{n+1} > 0$ $\frac{\sqrt{n}}{n+1} > \frac{\sqrt{n+1}}{n+2}$ Type: C Converges or Diverges: _______

5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+10}}$ $\lim_{n\to\infty} \frac{1}{\sqrt{n+10}} = 0$ $\frac{1}{\sqrt{n+10}} > 0$ $\frac{1}{\sqrt{n+10}} > \frac{1}{\sqrt{n+11}}$ Type: C Converges or Diverges: _______

6. $\sum_{n=0}^{\infty} 3^{n+2}2^{1-3n}$ Type: D Converges or Diverges: _______

$\frac{9(3^n)\cdot 2}{8^n}$ $r = \frac{3}{8}$

Part II: Name: ______________________

Use the ratio test to determine if each series converges or diverges. Show all work.

7. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$ $\displaystyle\lim_{n\to\infty}\frac{1}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{1}=\frac{(2n+1)!}{(2n+3)!}=\frac{1}{(2n+2)(2n+3)}$

$\displaystyle\lim_{n\to\infty}=0$ $\quad 0<1$ Converses

8. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n)^3}{2^{n+2}}$ $\displaystyle\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)^3}{2^{n+1+2}}\cdot\frac{2^{n+2}}{(-1)^n n^3}\right|=\frac{4(n^3+\cdots)}{8(n^3)}=\frac{1}{2}$

$\displaystyle\lim_{n\to\infty}=\frac{1}{2}<1$ Converses

9. $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{5n+1}$ $\displaystyle\lim_{n\to\infty}\frac{(2n+2)!}{5(n+1)+1}\cdot\frac{5n+1}{(2n)!}=\frac{(2n+1)(2n+2)(5n+1)}{5n+6}=\infty$

$\displaystyle\lim_{n\to\infty}=\infty>1$ diverges

Part III: Name: ____________________

Find the interval of convergence and radius of convergence for each geometric series. (You do not need to check endpoints)

10. $\sum_{n=0}^{\infty}\left(\frac{x-1}{2}\right)^n$

Interval: $-1 < x < 3$

Radius: 2

$-1 < \frac{x-1}{2} < 1$ $\quad -1 < x < 3$

$-2 < x-1 < 2$

11. $\sum_{n=0}^{\infty}\frac{(4x-3)^{3n}}{8^n}$

Interval: $\frac{1}{4} < x < \frac{5}{4}$

Radius: $\frac{1}{2}$

$-1 < \left(\frac{(4x-3)^3}{8}\right) < 1$

$-8 < (4x-3)^3 < 8$

$-2 < 4x-3 < 2$

$1 < 4x < 5$

$\frac{1}{4} < x < \frac{5}{4}$

Find the interval of convergence and radius of convergence for each series. (Hint: start with the ratio test) Remember to check endpoints, indicating the test used and showing all work.

12. $\sum_{n=0}^{\infty}\frac{(n+1)(x-2)^n}{(2n+1)!}$

Interval: $(-\infty, \infty)$

Radius: ∞

$$\frac{(n+1+1)(x-2)^{n+1}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{(n+1)(x-2)^n}$$

$$= (x-2)\left(\frac{(n+2)(2n+1)!}{(2n+3)!\,(n+1)}\right)$$

$$= (x-2)\left(\frac{(n+2)}{(2n+3)(2n+2)(n+1)}\right)$$

$$\lim_{n\to\infty}\frac{n+2}{(2n+3)(2n+2)(n+1)} = 0$$

$(-\infty, \infty)$

$r = \infty$

13. $\sum_{n=0}^{\infty}\frac{(6)^n(4x-1)^{n-1}}{n}$

Interval: $\frac{5}{24} \le x < \frac{7}{24}$

Radius: $\frac{1}{24}$

$$\frac{6^{n+1}(4x-1)^{n+1-1}}{n+1}\cdot\frac{n}{6^n(4x-1)^{n-1}}$$

$6(4x-1)\left(\frac{n}{n+1}\right)$ $\quad \lim_{n\to\infty}\frac{n}{n+1} = 1$

$-1 < 6(4x-1) < 1$

$5 < 24x < 7$

$\frac{5}{24} < x < \frac{7}{24}$

Check $x = \frac{5}{24}$

$\frac{6^n}{n}\left(-\frac{1}{6}\right)^{n-1}$

$\frac{6}{n}(-1)^{n-1}$

Con... - Alt

Check $x = \frac{7}{24}$

$\frac{6^n}{n}\left(\frac{1}{6}\right)^{n-1}$

$\frac{6^n}{n}\left(\frac{1}{6}\right)^n = \frac{1}{n}$

div - p-series $p = 1$

Part IV: Name: ____________________

Use the comparison test or the limit comparison test to determine if each series converges or diverges. Clearly show the series that you are using for comparison.

14. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

$b_n = \frac{n}{n^3} = \frac{1}{n^2}$ p-series $p = 2$ converges

$\frac{n}{n^3+1} < \frac{1}{n^2}$ so $\frac{n}{n^3+1}$ converges

15. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5^n}$

$b_n = \frac{1}{5^n}$ Geometric $r = \frac{1}{5}$ converges

$\frac{\cos n\pi}{5^n} < \frac{1}{5^n}$ so, $\frac{\cos n\pi}{5^n}$ converges

16. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 - \frac{1}{2}}$

$b_n = \frac{\sqrt{n}}{n^2} = \frac{1}{n^{\frac{3}{2}}}$ converges p-series $p = \frac{3}{2}$

$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n^2-\frac{1}{2}}}{\frac{\sqrt{n}}{n^2}} = \frac{n^2}{n^2-\frac{1}{2}} = 1$ $1 > 0$

both $\frac{\sqrt{n}}{n^2-\frac{1}{2}}$ and $\frac{\sqrt{n}}{n^2}$

have same behavior

both $\boxed{\text{converge}}$

Part V: Name: ____________________

Determine if each series converges or diverges
State the test you are using and show all work leading to your conclusion

17. $\sum_{n=5}^{\infty} \frac{6+8n+9n^2}{3+2n+n^2}$

$\lim_{n \to \infty} = 9 \neq 0$ nth Term Test

$\boxed{\text{diverges}}$

18. $\sum_{n=0}^{\infty} \frac{2^n \sin^2(5n)}{4^n + \cos^2 n}$

$b_n = \left(\frac{1}{2}\right)^n$ Geometric $r = \frac{1}{2}$ Converges

$$\frac{2^n \sin^2(5n)}{4^n + \cos^2(n)} < \left(\frac{1}{2}\right)^n$$

$\boxed{\text{Converges}}$

19. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{7+2n}$

$\lim_{n \to \infty} \frac{1}{7+2n} = 0$ $\quad \frac{1}{7+2n} > 0$ $\quad \frac{1}{7+2(n+1)} < \frac{1}{7+2n}$

Alternating Series Test $\boxed{\text{Converges}}$

20. $\sum_{n=3}^{\infty} \frac{e^{4n}}{(n-2)!}$

$$\lim_{n \to \infty} \frac{e^{4(n+1)}}{(n+1-2)!} \cdot \frac{(n-2)!}{e^{4n}} = \frac{e^4}{n-1} \quad \lim_{n \to \infty} = 0$$

$0 < 1$ $\boxed{\text{Converges}}$