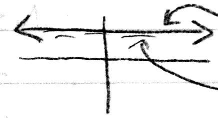
II Derivative Rules (2C)

A. The derivative of a constant

$\boxed{\frac{d}{dx}(c)=0}$ or $\boxed{\text{If } f(x)=c \text{ , then } f'(x)=0}$

$\boxed{\text{ex1}}$ $\frac{d}{dx}(4)=0$

$\boxed{\text{ex2}}$ If $f(x)=5^k$, where k is a constant, $f'(x)=0$

Visual: 

≥ proof ≤ If $f(x)=c$, using the definition of the derivative:

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

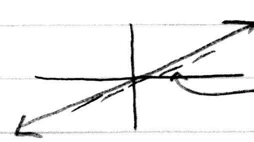
$$\lim_{h\to 0}\frac{c-c}{h}=\lim_{h\to 0}\frac{0}{h}=\lim_{h\to 0} 0=0 \quad \text{AWAD!}$$

B. The derivative of a linear function

$\boxed{\frac{d}{dx}(cx+b)=c}$ or $\boxed{\text{If } f(x)=cx+b \text{ , } f'(x)=c}$

$\boxed{\text{ex3}}$ If $f(x)=22x+5$, $f'(x)=22$

$\boxed{\text{ex4}}$ If $f(x)=\frac{1}{2}x$, $f'(x)=\frac{1}{2}$

Visual: 

≥ proof ≤ If $f(x)=cx+b$

$$\lim_{h\to 0}\frac{c(x+h)+b-(cx+b)}{h}=\lim_{h\to 0}\frac{\cancel{cx}+ch+\cancel{b}-\cancel{cx}-\cancel{b}}{h}$$

$$=\lim_{h\to 0}\frac{ch}{h}=\lim_{h\to 0} c=c. \quad \text{AWAD!}$$

E. The Derivative of a Product

$$\boxed{\left[f(x)\cdot g(x)\right]' = f(x)\cdot g'(x) + g(x)\cdot f'(x)}$$

$$\boxed{\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)}$$

$\boxed{\text{ex 9}}$ $f(x) = \underset{\uparrow\, 1^{st}}{(x^3+5x+7)}\underset{\uparrow\, 2^{nd}}{(x^3+6x+8)}$

$$f'(x) = (x^3+5x+7)(x^3+6x+8)' + (x^3+6x+8)(x^3+5x+7)'$$

$$= \boxed{(x^3+5x+7)(3x^2+6) + (x^3+6x+8)(3x^2+5)}$$

proof: $(f(x)\cdot g(x))'$

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$=0$ ninja

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) \overbrace{- f(x+h)g(x) + f(x+h)g(x)} - f(x)g(x)}{h}$$

$$\lim_{h\to 0} \frac{f(x+h)\left[g(x+h) - g(x)\right] + g(x)\left[f(x+h) - f(x)\right]}{h}$$

$$\lim_{h\to 0} \underbrace{f(x+h)}_{\Downarrow\ f(x)}\ \underbrace{\frac{\left[g(x+h) - g(x)\right]}{h}}_{\Downarrow\ \text{derivative of } g(x)} + \lim_{h\to 0} \underbrace{g(x)}_{\Downarrow\ g(x)}\ \underbrace{\frac{\left[f(x+h) - f(x)\right]}{h}}_{\Downarrow\ \text{derivative of } f(x)}$$

$$= f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

AWAD!

F. The Derivative of a Quotient

$$\boxed{\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}}$$

$$\boxed{\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}}$$

ex 10 $f(x) = \frac{x-3}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(x-3)' - (x-3)(x^2+1)'}{(x^2+1)^2} = \frac{(x^2+1)(1) - (x-3)(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2+6x}{(x^2+1)^2} = \boxed{\frac{-x^2+6x+1}{(x^2+1)^2}}$$

proof: $\frac{f(x)}{g(x)}$

$$\lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x)g(x+h)}$$

=0 Ninja!

$$\lim_{h\to 0} \frac{f(x+h)g(x) \underbrace{- f(x)g(x) + f(x)g(x)}_{=0} - f(x)g(x+h)}{h\,g(x)g(x+h)}$$

$$\lim_{h\to 0} \frac{\overbrace{g(x)}^{\Rightarrow g(x)}\overbrace{\boxed{f(x+h)-f(x)}}^{\Rightarrow f'(x)}}{h\,\underbrace{g(x)}_{\Downarrow g(x)}\underbrace{g(x+h)}_{\Downarrow g(x)}} + \lim_{h\to 0} \frac{\overbrace{-f(x)}^{\Rightarrow -f(x)}\overbrace{\boxed{g(x+h)-g(x)}}^{\Rightarrow g'(x)}}{h\,\underbrace{g(x)}_{\Downarrow g(x)}\underbrace{g(x+h)}_{\Downarrow g(x)}}$$

$$\frac{g(x)\cdot f'(x)}{(g(x))^2} + \frac{-f(x)\cdot g'(x)}{(g(x))^2} = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{[g(x)]^2}$$

AWAD!

G. Extensions

1. Combination

quotient rule

$$\frac{d}{dx}\left(\frac{e^x x^2+1}{3x}\right) = \frac{(3x)(e^x x^2+1)' - (e^x x^2+1)(3x)'}{(3x)^2}$$

product rule

$$\frac{(3x)(e^x \cdot 2x + x^2 \cdot e^x) - (e^x x^2+1)(3)}{9x^2} = \frac{6e^x x^2 + 3x^3 e^x - 3e^x x^2 - 3}{9x^2}$$

$$= \frac{3e^x x^2 + 3e^x x^3 - 3}{9x^2} = \boxed{\frac{e^x x^2 + e^x x^3 - 1}{3x^2}}$$

2. Extension of the Product Rule

$$\boxed{\left[f(x)\cdot g(x)\cdot h(x)\right]' = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x)}$$

3. Derivatives to Memorize

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

memorize for Speed!

4. The Derivative at a Point

$$\frac{d}{dx}\left(e^x(3x^2 - x + 1)\right)\Big|_{x=1}$$

means evaluate the derivative at $x=1$

this will find the slope of the tangent line at $x=1$.

$$e^x(6x-1) + (3x^2 - x + 1)e^x \Big|_{x=1}$$

$$e^1(6(1)-1) + (3(1)^2 - (1) + 1)e^1$$

$$5e + 3e = \boxed{8e}$$