8/21

II Calculating Limits Using Limit Theorems (1D)

- ☆ A picture is <u>not</u> proof.
- ☆ Inductive reasoning is <u>not</u> proof {tables}
  - → these are estimations of the limit
- ☆ we need theorems to prove :)

A. Limit Theorems

"plug in rules"

1. $\lim_{x \to a} x = a$
2. $\lim_{x \to a} x^n = a^n$ {n is a positive integer}
3. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ {n is a positive integer, and if n is even then $a > 0$}
4. $\lim_{x \to a} c = c$ {where c is a constant}
5. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
6. $\lim_{x \to a} [cf(x)] = c \cdot \lim_{x \to a} f(x)$
7. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
8. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ {$\lim_{x \to a} g(x) \neq 0$}
9. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$ {n is a positive integer}
10. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ {n is a positive integer and if n is even $\lim_{x \to a} f(x) > 0$}

ex1 Justify the limit is the indicated number.

$$\lim_{x\to 1} \frac{x(x+3)}{x+1} = 2$$

$$\lim_{x\to 1} \frac{x(x+3)}{x+1} \overset{\#8}{=} \frac{\lim_{x\to 1} x(x+3)}{\lim_{x\to 1}(x+1)} \overset{\#7}{=} \frac{\left[\lim_{x\to 1} x\right]\cdot\left[\lim_{x\to 1}(x+3)\right]}{\lim_{x\to 1}(x+1)} =$$

$$\overset{\#5}{=} \frac{\lim_{x\to 1} x \cdot \left[\lim_{x\to 1} x + \lim_{x\to 1} 3\right]}{\lim_{x\to 1} x + \lim_{x\to 1} 1} = \frac{\overset{\#1}{1} \cdot \left[\overset{\#1}{1} + \overset{\#4}{3}\right]}{\underset{\#1}{1} + \underset{\#4}{1}} = \frac{4}{2} = 2$$

B. Justify Vs. Evaluate

1. Justify the limit:

#5   #6   #2 #1 #4

$$\lim_{x\to6}\frac{x^2-2x+1}{x} = \frac{\lim_{x\to6}x^2 - 2\lim_{x\to6}x + \lim_{x\to6}1}{\lim_{x\to6}x} = \frac{(6)^2-2(6)+1}{6}$$

#8   #1

$$= \frac{25}{6}$$

2. Evaluate the limit

b/c limit theorems allow this!

$$\lim_{x\to6}\frac{x^2-2x+1}{x} = \frac{6^2-2(6)+1}{6} = \boxed{\frac{25}{6}}$$

Note: $\lim_{x\to6} f(x) \neq f(6)$ two different ideas...

a. what if no limit theorems work?
→ use graph/table/calculator

b. if you can use limit theorems...
→ "plug in" and simplify

<u>HOWEVER</u>... you may get stuck!

if $\lim_{x\to0} f(x) = \frac{0}{\#} = 0$ good.

if $\lim_{x\to0} f(x) = \frac{\#}{0} = \infty$ $\therefore$ DNE   good.
approaching!

if $\lim_{x\to0} f(x) = \frac{0 \text{ approaching}}{0 \text{ approaching}} = ???$ ... it's <u>not</u> DNE or 0 ...
IDK
for now 🙂

C. Evaluating Limits of Indeterminate Form (IE)

If you use the Limit Theorems to "plug in" and get:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^0, $0\cdot\infty$, 1^∞, ∞^0, $\infty \pm \infty$

this is called <u>indeterminate form</u>, Solve by:

* using algebra rules to simplify / change the format
  → factor / cancel / rationalize / trig IDs / etc
* try limit theorems again
* repeat as necessary

ex 1 $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4}-2}{x}$ can't "plug in" w/ L.T. $\frac{0}{0}$ ∴ indeterminate form

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x\to 0} \frac{x+4-4}{x\sqrt{x+4}+2x} =$$

$$\lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}\sqrt{x+4}+2\cancel{x}} = \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

* or use your calculator

f3 → limit → limit(f(x), x, a)

$$\text{limit}\left(\frac{(\sqrt{x+4}-2)}{x}, x, 0\right)$$