AP Calculus

REVIEW FOR 3rd QUARTER MIDTERM

Name___________________________________

Seat # ________ Date _____________________

Areas and Volumes

UNLESS INDICATED, DO NOT USE YOUR CALCULATOR FOR ANY OF THESE QUESTIONS

In questions 1-3, make a sketch of each region and the representative rectangle and answer the question.

1. Find the area of the region enclosed by the graphs of $y = x^2$ and $y = \sqrt{x}$.

2. Find the area of the region enclosed by the graphs of $x = y^2$ and $y = x - 2$.

3. Find the total area of the region(s) enclosed by the graphs of $y = x^3 - 3x^2 - x + 3$ and $y = x^2 - 5$. You may use your graphing calculator.

In questions 4-10, make a sketch of each region and the representative rectangle. Then answer the questions.

4. Find the volume of the solid generated when the region enclosed by the graphs of the equations $y = x^2 + 1$, $y = x$, $x = 0$, and $x = 2$ is revolved **about the *x*-axis**. You may use your graphing calculator.

5. Find the volume of the solid generated when the region enclosed by the graphs of $y = \sqrt{4 - x^2}$, $y = x$, and the y-axis is revolved **about the *y*-axis.** You may use your graphing calculator.

6. Find the volume of the solid generated when the region enclosed by the graphs of $y = \sqrt{x + 2}$, $y = x$, and the x-axis is revolved **about the *x*-axis.**

7. Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 9$ is revolved **about the line *x* = 9.**

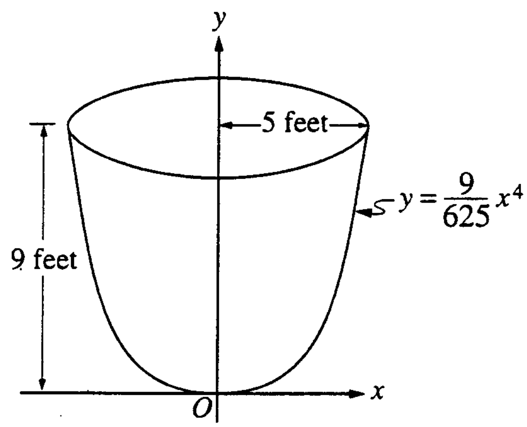
8. Find the volume of the solid that results when the region in exercise 7 is revolved **about the line *y* = 4.** You may use your graphing calculator.

9. Find the volume of a solid whose base is the region bounded by the graphs of $y = e^x$ and $y = x + 4$, and whose cross sections **perpendicular to the *x*-axis** are equilateral triangles. You may use your graphing calculator.

10. Find the volume of the solid described in exercise 9 if the cross sections are equilateral triangles **perpendicular to the *y*-axis**. You may use your graphing calculator.

SEE OTHER SIDE

ESSAY FROM 1996 AB EXAM: (graphing calculator may be used)



11. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.
    a) Use a definite integral to find the volume of the tank. Indicate units of measure.
    b) To the nearest minute, how long would it take to fill the tank if the tank was initially empty?
    c) (*Challenge!*) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.

Just for fun (again!)

No more proofs (I know by now you do believe that 2 = 1…)

You are going to use your graphing calculator, to graph two functions and find the area between their curves. To get a good picture of the graph, set your window screen to the following parameters:

Xmin = –1.8
Xmax = 1.8
Xscl = 0
Ymin = –2.2
Ymax = 2.2
Yscl = 0
Xres = 1

The functions are $f(x) = |x| - \sqrt{1 - x^2}$ and $g(x) = |x| + \sqrt{1 - x^2}$. Use your calculator to find the area between the two curves. Do you recognize the value of the area?

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AREAS AND VOLUMES

ANSWER KEY

Areas and Volumes

1. $\int_0^1 \left(\sqrt{x} - x^2\right) \cdot dx = \frac{1}{3}$.

2. $2\int_0^1 \sqrt{x} \cdot dx + \int_1^4 \left(\sqrt{x} - (x-2)\right) \cdot dx = \frac{4}{3} + \frac{19}{6} = \frac{9}{2}$ or $\int_{-1}^2 \left[(y+2) - y^2\right] \cdot dy = \frac{9}{2}$

3. $\left(x^3 - 3x^2 - x + 3\right) = \left(x^2 - 5\right) \Rightarrow x \approx -1.323;\ 1.642;\ 3.681$

   $\int_{-1.323}^{1.642} \left[\left(x^3 - 3x^2 - x + 3\right) - \left(x^2 - 5\right)\right] \cdot dx + \int_{1.642}^{3.681} \left[\left(x^2 - 5\right) - \left(x^3 - 3x^2 - x + 3\right)\right] \cdot dx \approx 15.308 + 5.633 \approx 20.941$

4. $\pi\int_0^2 \left[\left(x^2 + 1\right)^2 - x^2\right] \cdot dx = \frac{166\pi}{15}$

5. $2\pi\int_0^{\sqrt{2}} x \cdot \left(\sqrt{4 - x^2} - x\right) \cdot dx = \frac{16 - 8\sqrt{2}}{3}\pi \approx 4.907$ or $\pi\int_0^{\sqrt{2}} (y)^2\, dy + \pi\int_{\sqrt{2}}^2 \left(\sqrt{4 - y^2}\right)^2 dy = 4.907$

6. $2\pi\int_0^2 y \cdot \left(y - \left(y^2 - 2\right)\right) \cdot dy = \frac{16}{3}\pi \approx 16.755$ or $\pi\int_{-2}^0 \left(\sqrt{x+2}\right)^2 dx + \pi\int_0^2 \left[\left(\sqrt{x+2}\right)^2 - x^2\right] \cdot dx = 16.755$

7. $2\pi\int_0^9 (9 - x) \cdot \sqrt{x} \cdot dx = \frac{648}{5}\pi \approx 407.150$ or $\pi\int_0^3 \left(9 - y^2\right)^2 dy \approx 407.150$

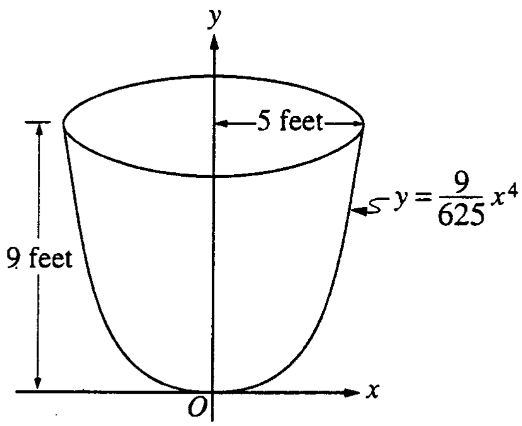
8. $\pi\int_0^9 \left[4^2 - \left(4 - \sqrt{x}\right)^2\right] \cdot dx = \frac{207}{2}\pi \approx 325.155$ or $2\pi\int_0^3 (4 - y) \cdot \left(9 - y^2\right) \cdot dy \approx 325.155$

9. Intersections: (–3.981, 0.019) and (1.749, 5.749)

   $\int_{-3.981}^{1.749} \frac{\sqrt{3}}{4} \cdot \left((x+4) - e^x\right)^2 \cdot dx \approx 10.921$

10. $\int_{0.019}^{5.749} \frac{\sqrt{3}}{4} \cdot \left(\ln y - (y - 4)\right)^2 \cdot dy \approx 10.921$

ESSAY FROM 1996 AB EXAM: (graphing calculator may be used)



11. a) $V = \pi \int_0^9 \left(\sqrt[4]{\frac{625y}{9}} \right)^2 dy = 150\pi \approx 471.239$ cubic ft or $V = 2\pi \int_0^5 x \cdot \left(9 - \frac{9}{625} x^4 \right) \cdot dx \approx 471.239$

b) $471.239/8 \approx 58.904 \Rightarrow 59$ minutes

c) $V_{WATER} = \pi \int_0^h \left(\sqrt[4]{\frac{625y}{9}} \right)^2 dy$

d) Taking derivatives:

$$\frac{d}{dt}\left(V_{WATER} = \pi \int_0^h \left(\sqrt[4]{\frac{625y}{9}} \right)^2 dy \right) \Rightarrow \frac{dV_{WATER}}{dt} = \pi \left(\sqrt[4]{\frac{625h}{9}} \right)^2 \frac{dh}{dt} = \pi \sqrt{\frac{625h}{9}} \cdot \frac{dh}{dt}$$

Replacing values: $\left. \begin{matrix} \frac{dV_{WATER}}{dt} = 8 \\ h = 4 \end{matrix} \right\} \Rightarrow 8 = \pi \sqrt{\frac{625 \cdot 4}{9}} \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{12}{25\pi} = 0.153$ ft/minute