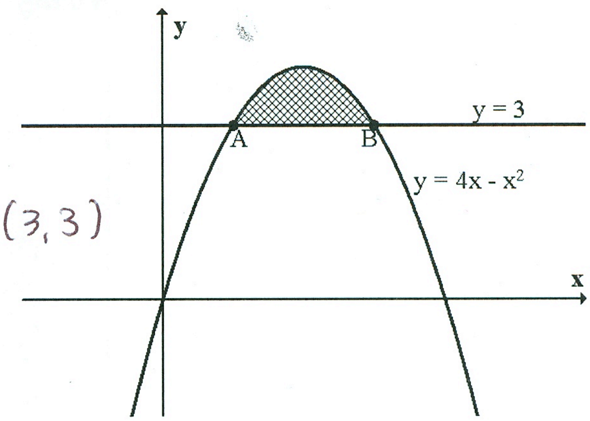
Area Between two Curves



1. The diagram opposite shows the curve $y = 4x - x^2$ and the line $y = 3$.

   (a) Find the coordinates of A and B. $(1,3)\ (3,3)$

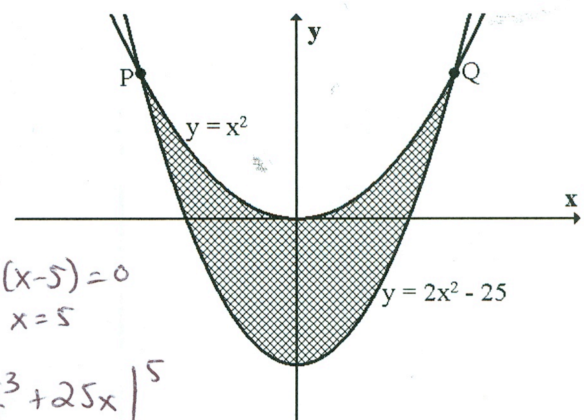
   (b) Calculate the shaded area.

a) $3 = 4x - x^2 \quad x^2 - 4x + 3 = 0 \quad (x-1)(x-3)$

$x = 1 \quad x = 3$

b) $\int_1^3 4x - x^2 - 3\,dx = 2x^2 - \frac{x^3}{3} - 3x \Big|_1^3$

$(18 - 9 - 9) - (2 - \frac{1}{3} - 3) = \boxed{1\frac{1}{3}}$



2. The curves with equations $y = x^2$ and $y = 2x^2 - 25$ intersect at P and Q.

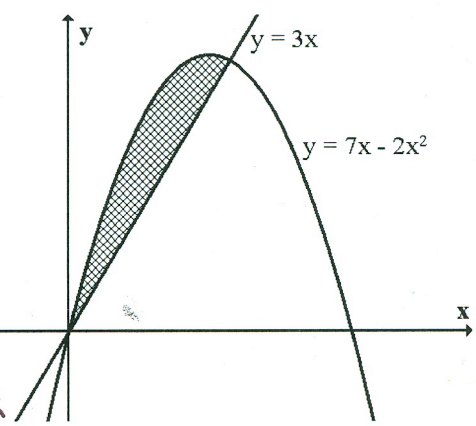
   Calculate the area enclosed between the curves.

$x^2 = 2x^2 - 25 \quad 0 = x^2 - 25 \quad (x+5)(x-5) = 0$

$x = -5 \quad x = 5$

$\int_{-5}^{5} x^2 - (2x^2 - 25)\,dx = \int_{-5}^{5} -x^2 + 25\,dx = -\frac{x^3}{3} + 25x \Big|_{-5}^{5}$

$\left(\frac{-125}{3} + 125\right) - \left(\frac{125}{3} - 125\right) = \boxed{166.667}$



3. The diagram opposite shows the curve $y = 7x - 2x^2$ and the line $y = 3x$.

   Calculate the shaded area.

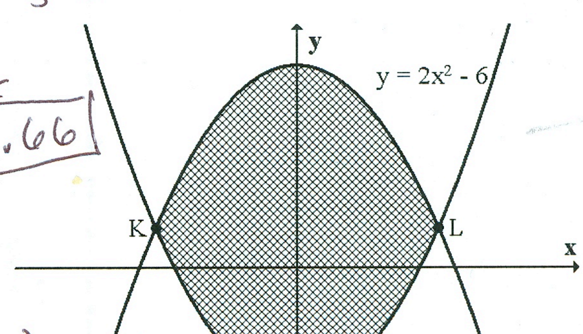
$3x = 7x - 2x^2 \quad 2x^2 - 4x = 0 \quad x(x-2) = 0$

$x = 0 \quad x = 2$

$\int_0^2 7x - 2x^2 - 3x\,dx = \int_0^2 4x - 2x^2\,dx = 2x^2 - \frac{2x^3}{3} \Big|_0^2$

$\cancel{\left(32 - \frac{128}{3}\right)}$ $\left(8 - \frac{16}{3}\right) - 0 =$

$\boxed{2.66}$



4. The curves with equations $y = 2x^2 - 6$ and $y = 10 - 2x^2$ intersect at K and L.

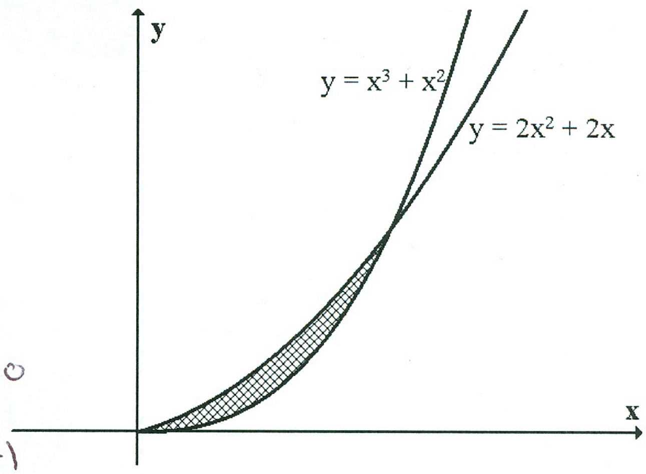
   Calculate the area enclosed by these two curves.

$10 - 2x^2 = 2x^2 - 6 \quad 4x^2 - 16 = 0 \quad 4(x^2 - 4)$

$x = \pm 2$

$\int_{-2}^{2} (2x^2 - 6) - (10 - 2x^2)\,dx = \int_{-2}^{2} 4x^2 - 16\,dx = \frac{4x^3}{3} - 16x \Big|_{-2}^{2}$

$\left(\frac{4(8)}{3} - 16(2)\right) - \left(\frac{-4(8)}{3} + 16(2)\right) = \boxed{42.66}$



5. The diagram opposite shows part of the curves $y = x^3 + x^2$ and $y = 2x^2 + 2x$.

   Calculate the shaded area.

$x^3 + x^2 = 2x^2 + 2x \quad x^3 - x^2 - 2x = 0$

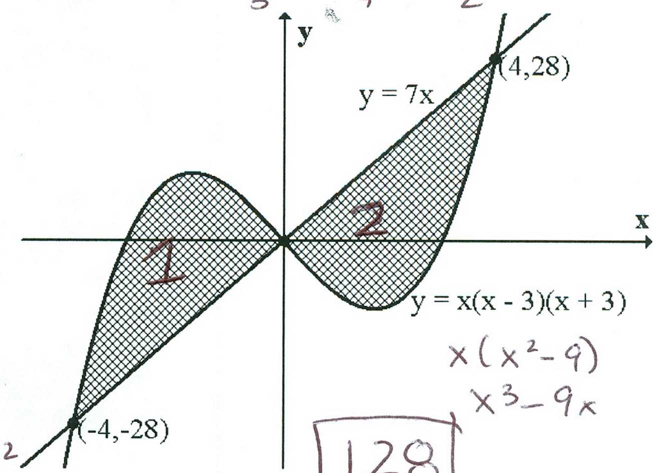
$x(x^2 - x - 2) = 0$

$x(x-2)(x+1)$

$x = 0 \quad x = 2 \quad x = -1$

$$\int_0^2 2x^2 + 2x - (x^3 + x^2)\,dx = \int_0^2 x^2 - x^3 + 2x = \frac{x^3}{3} - \frac{x^4}{4} + \frac{2x^2}{2}\Big|_0^2$$

$$\frac{8}{3} - \frac{16}{4} + \frac{8}{2} = \boxed{\frac{8}{3}}$$



6. The curve $y = x(x-3)(x+3)$ and the line $y = 7x$ intersect at the points (0,0), (-4,-28) and (4,28).

   Calculate the area enclosed by the curve and the line.

$x(x^2 - 9)$

$x^3 - 9x$

$\boxed{128}$

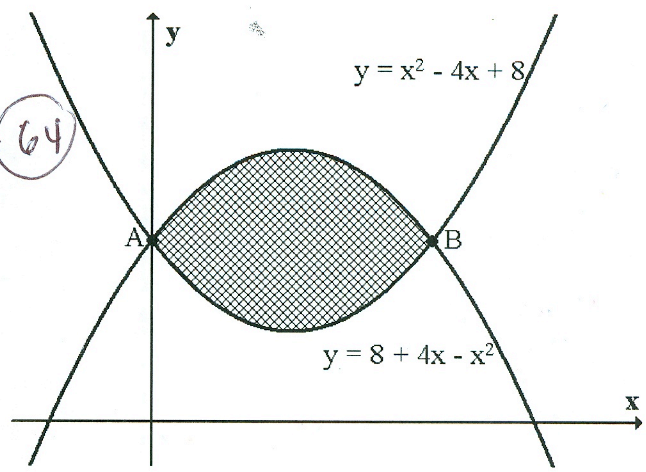
Region 1

$$\int_{-4}^{0} x^3 - 9x - 7x\,dx = \int_{-4}^{0} x^3 - 16x = \frac{x^4}{4} - 8x^2$$

$$0 - \left(\frac{(-4)^4}{4} - 8(16)\right) = 0 - (-64) = 64$$

Region 2

$$\int_0^4 7x - (x^3 - 9x) = \int_0^4 16x - x^3 = 8x^2 - \frac{x^4}{4}\Big|_0^4 = 64$$



7. The parabolas $y = x^2 - 4x + 8$ and $y = 8 + 4x - x^2$ intersect at A and B.

   (a) Find the coordinates of A and B.
   (b) Calculate the shaded area.

$x^2 - 4x + 8 = 8 + 4x - x^2$

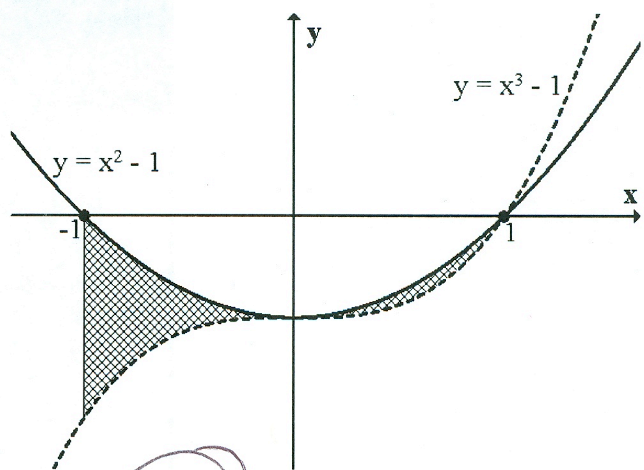
$2x^2 - 8x = 0 \quad 2x(x-4) = 0$

$x = 0$

$x = 4$

$$\int_0^4 8 + 4x - x^2 - (x^2 - 4x + 8)\,dx = \int_0^4 8x - 2x^2 = 4x^2 - \frac{2x^3}{3}\Big|_0^4$$

$$\left(64 - \frac{128}{3}\right) - 0 = \boxed{21.33}$$

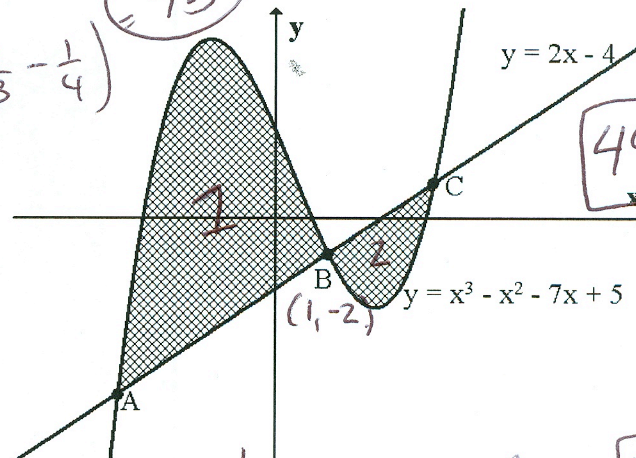


8. The diagram shows parts of the curves $y = x^3 - 1$ and $y = x^2 - 1$.

Calculate the shaded area.

$$\int_{-1}^{1} x^2-1-(x^3-1)\,dx = \int_{-1}^{1} x^2-x^3 =$$

$$\frac{x^3}{3}-\frac{x^4}{4}\Big|_{-1}^{1} = \left(\frac{1}{3}-\frac{1}{4}\right)-\left(-\frac{1}{3}-\frac{1}{4}\right) = 2/3$$



$\boxed{49.33}$

9. The curve $y = x^3 - x^2 - 7x + 5$ and the line $y = 2x - 4$ are shown opposite.

(a) B has coordinates (1,-2). Find the coordinates of A and C.
(b) Hence calculate the shaded area.

$x^3-x^2-7x+5 = 2x-4$

$x^3-x^2-9x+9 = 0$

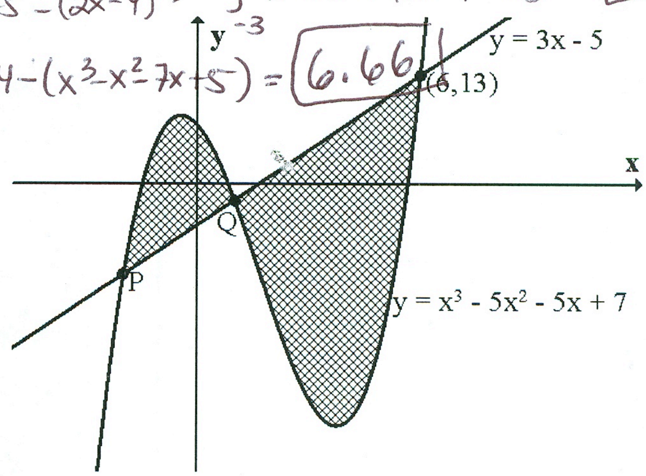
$x^2(x-1) -9(x-1)$

$(x^2-9)(x-1) = 0$

$x = \pm 3 \quad x = 1$

(1) $\int_{-3}^{1} x^3-x^2-7x+5-(2x-4) = \int_{-3}^{1} x^3-x^2-9x+9 = \boxed{42.66}$

(2) $\int_{1}^{3} 2x-4-(x^3-x^2-7x+5) = \boxed{6.66}$



10. The diagram shows the line $y = 3x - 5$ and the curve $y = x^3 - 5x^2 - 5x + 7$.

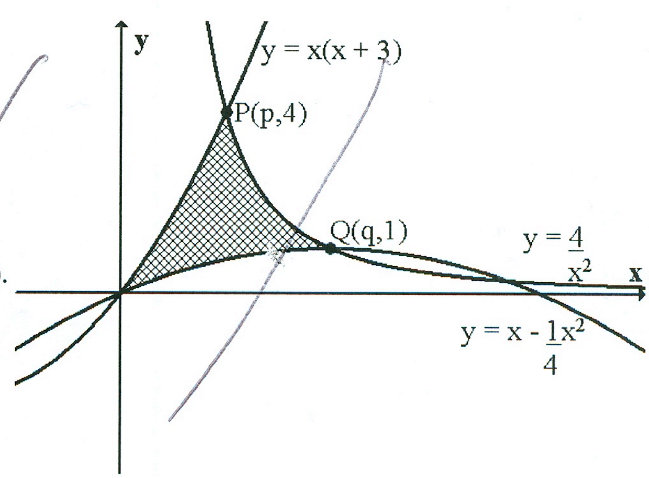
(a) Find the coordinates of P and Q.
(b) Calculate the shaded area.

SKIP

$3x-5 = x^3-5x^2-5x+7$

$0 = x^3-5x-8x+12$

$x(x^2-5) -$



11. The diagram opposite shows an area enclosed by 3 curves:

SKIP

$y = x(x + 3)$, $y = \dfrac{4}{x^2}$ and $y = x - \dfrac{1}{4}x^2$

(a) P and Q have coordinates (p,4) and (q,1). Find the values of p and q.

(b) Calculate the shaded area.

Durbec

CALCULUS

Practice Quiz-Area Between Curves

Practice Quiz:

For Each of the following; sketch the graphs and label points of intersection, find the area of the region bounded by the curves.

1. $f(x) = x^2 + 2x + 1$
   $g(x) = 2x + 5$

2. $f(x) = 2 - x^2$
   $g(x) = x$

$$\int_{-2}^{1} (-x^2+2) - x)\,dx =$$

$x^2 + x - 2$

$(x+2)(x-1)$

$$\frac{-x^3}{3} - \frac{x^2}{2} + 2x \Big|_{-2}^{1}$$

4.5

$$\left(\frac{-1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$

1.166 − (-3.333

3. $f(x) = x^3 - 3x^2 + 3x$
   $g(x) = x^2$

$x^3 - 4x^2 + 3x = 0$

$x(x^2 - 4x + 3) = 0$

$(x-1)(x-3)$

$x=0 \quad x=1 \quad x=3$

$$\int_0^1 x^3 - 4x^2 + 3x =$$

$$\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2} \Big|_0^1$$

.41666

$$\int_1^3 -x^3 + 4x^2 - 3x$$

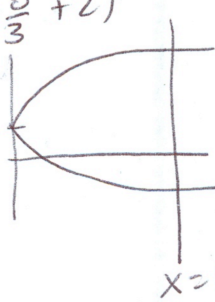
$$-\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} \Big|_1^3$$

5.333

10.66

$$\left(10 - \frac{8}{3} - 2\right) - \left(-10 + \frac{8}{3} + 2\right)$$

4. $x = y^2 + 1$
   $x = 5$



x=5

$$5y - \frac{y^3}{3} - y \Big|_{-2}^{2}$$

$$\int_{-2}^{2} 5 - (y^2+1)$$

$\frac{-81}{4} + 36 - 13.5$

2.25

3.08