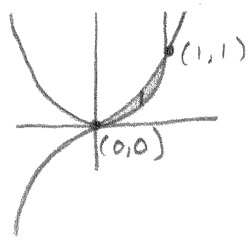
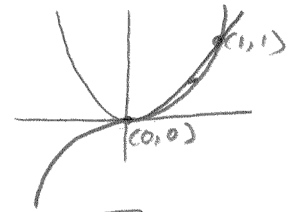
AP Calculus
Miss Brown

Area Between Curves Practice

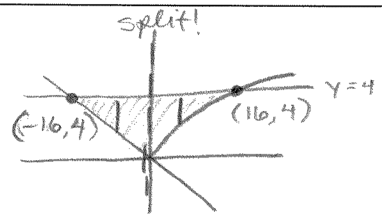
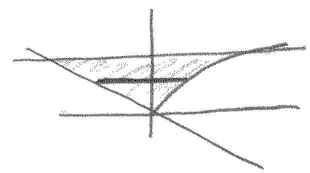
Name Answer Key
Date

Directions: **For #1-12, find the area of each bounded region using both a vertical and a horizontal cross-section.**

1. Find the area of the region bounded by $y = x^2$ and $y = x^3$.

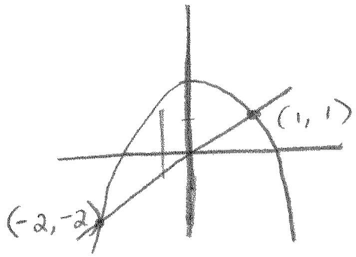
VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $A = \int_0^1 (x^2 - x^3)\,dx$ $A = \frac{1}{12}$	 $\boxed{\begin{array}{l} x = \sqrt{y} \\ x = \sqrt[3]{y} \end{array}}$ $A = \int_0^1 (\sqrt[3]{y} - \sqrt{y})\,dy$ $A = \frac{1}{12}$

Typo! fixed 2. Find the area of the region bounded by $y = \sqrt{x}$ and $y = -\frac{1}{4}x$, $y = 4$

VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $A = \int_{-16}^{0} (4 + \frac{1}{4}x)\,dx + \int_0^{16} (4 - \sqrt{x})\,dx$ $32 \quad + \quad 21.333$ $A = \frac{160}{3}$	 $\boxed{\begin{array}{l} x = y^2 \\ x = -4y \end{array}}$ $A = \int_0^4 (y^2 + 4y)\,dy$ $A = \frac{160}{3}$

3. Find the area of the region bounded by $y = 2 - x^2$ and $y = x$.

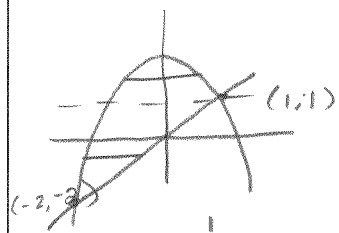
VERTICAL CROSS-SECTION



$$A = \int_{-2}^{1} \left((2 - x^2) - (x) \right) dx$$

$$A = \frac{9}{2}$$

HORIZONTAL CROSS-SECTION



$$\boxed{\begin{array}{l} x = \pm\sqrt{2-y} \\ x = y \end{array}}$$

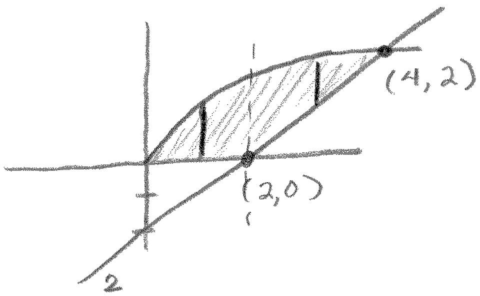
$$A = \int_{-2}^{1} \left(y + \sqrt{2-y} \right) dy + \int_{1}^{2} \left(\sqrt{2-y} + \sqrt{2-y} \right) dy$$

3.166666 + 1.3333

$$A = \frac{9}{2}$$

4. Find the area of the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x - 2$.

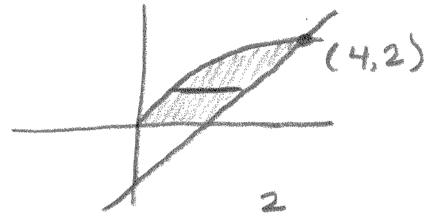
VERTICAL CROSS-SECTION



$$A = \int_{0}^{2} \left(\sqrt{x} - 0 \right) dx + \int_{2}^{4} \left(\sqrt{x} - (x - 2) \right) dx$$

$$A = \frac{10}{3}$$

HORIZONTAL CROSS-SECTION



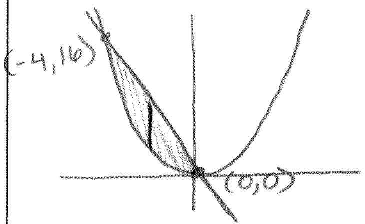
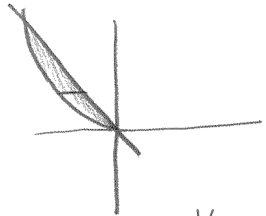
$$\boxed{\begin{array}{l} x = y + 2 \\ x = y^2 \end{array}}$$

$$A = \int_{0}^{2} \left((y + 2) - y^2 \right) dy$$

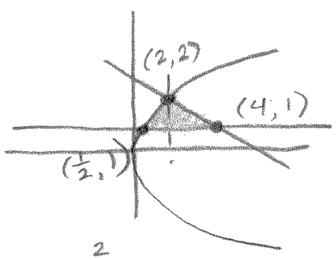
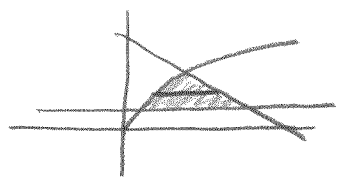
$$A = \frac{10}{3}$$

~~TF~~ Fixed!

5. Find the area of the region bounded by $y = x^2$, ~~$y = 4$~~, and $y = -4x$.

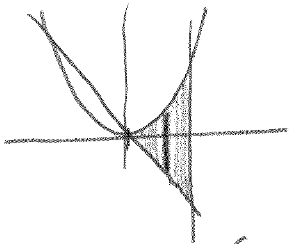
VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $A = \int_{-4}^{0} (-4x - x^2)\,dx$ $A = \frac{32}{3}$	 $\boxed{\begin{array}{l} x = -\frac{1}{4}y \\ x = \pm\sqrt{y} \end{array}}$ $A = \int_{0}^{16} \left(-\frac{1}{4}y + \sqrt{y}\right)dy$ $A = \frac{32}{3}$

6. Find the area of the region bounded by $x = \frac{1}{2}y^2$, $y = 1$, and $y = -\frac{1}{2}x + 3$.

VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $\boxed{y = \pm\sqrt{2x}}$ $A = \int_{\frac{1}{2}}^{2} (\sqrt{2x} - 1)\,dx + \int_{2}^{4} \left(-\frac{1}{2}x + 3 - 1\right)dx$ $A = \frac{11}{6}$	 $\boxed{\begin{array}{l} x = -2(y-3) \\ x = \frac{1}{2}y^2 \end{array}}$ $A = \int_{1}^{2} \left(-2(y-3) - \frac{1}{2}y^2\right)dy$ $A = \frac{11}{6}$

7. AP QUESTION -- No Calculator -- Find the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$.

VERTICAL CROSS-SECTION



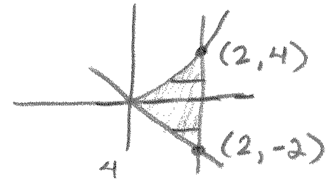
$$A = \int_0^2 (x^2 + x)\,dx$$

$$\left.\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2$$

$$\left(\frac{8}{3} + 2\right) - (0)$$

$$\frac{8}{3} + \frac{6}{3} = \frac{14}{3}$$

HORIZONTAL CROSS-SECTION



$$\boxed{\begin{array}{l} x = \pm\sqrt{y} \\ x = -y \end{array}}$$

$$A = \int_0^4 (2 - \sqrt{y})\,dy + \int_{-2}^0 (2 + y)\,dy$$

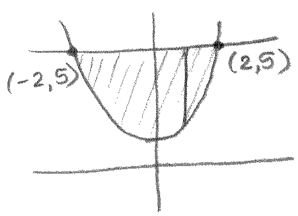
$$\left.2y - \frac{2}{3}y^{\frac{3}{2}}\right]_0^4 + \left.2y + \frac{y^2}{2}\right]_{-2}^0$$

$$\left(8 - \frac{16}{3}\right) - (0) + (0) - (-4 + 2)$$

$$\frac{24}{3} - \frac{16}{3} + 2 \quad \frac{8}{3} + \frac{6}{3} = \frac{14}{3}$$

8. AP QUESTION -- No Calculator -- Find the area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$.

VERTICAL CROSS-SECTION



$$A = \int_{\cancel{-2}}^{2} (5 - (x^2 + 1))\,dx$$

$$A = \int_{-2}^{2} (5 - x^2 - 1)\,dx$$

$$A = \int_{-2}^{2} (4 - x^2)\,dx$$

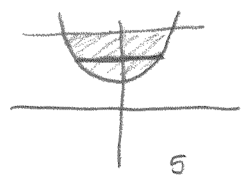
$$\left.4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$16 - \frac{16}{3}$$

$$\frac{48}{3} - \frac{16}{3} = \frac{32}{3}$$

HORIZONTAL CROSS-SECTION



$$\boxed{x = \pm\sqrt{y - 1}}$$

$$A = \int_1^5 \left(\sqrt{y-1} + \sqrt{y-1}\right) dy$$

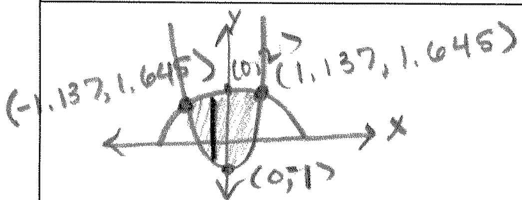
$$A = 2\int_1^5 \sqrt{y-1}\,dy \qquad u = y - 1 \quad \frac{du}{dy} = 1$$

$$A = 2\int_0^4 u^{\frac{1}{2}}\,du$$

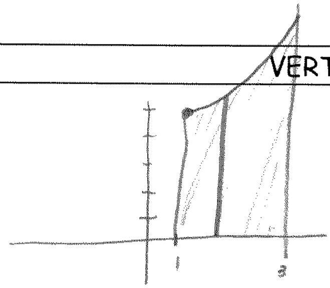
$$\left.\frac{2}{3}u^{\frac{3}{2}}\right]_0^4$$

$$\frac{4}{3}(8 - 0) = \frac{32}{3}$$

9. AP QUESTION -- *Calculator* -- Find the area of the region enclosed by the graphs of $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$.

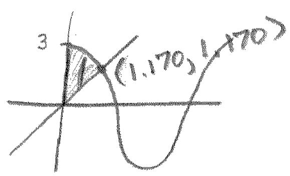
VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $A = \int T - B$ $A = \int_{-1.137}^{1.137} \left(\sqrt{4-x^2} - (e^{x^2} - 2)\right) dx = 5.050$	$A = \int R - L$ $A = \int_{1.645}^{2} \left(\sqrt{4-y^2} + \sqrt{4-y^2}\right) dy$ + $\int_{-1}^{1.645} \left(\sqrt{\ln(y+2)} + \sqrt{\ln(y+2)}\right) dy$ $A = 5.050$ $\boxed{\begin{aligned} e^{x^2} &= y + 2 \\ x^2 &= \ln(y+2) \\ x &= \pm\sqrt{\ln(y+2)} \\ x^2 &= 4 - y^2 \\ x &= \pm\sqrt{4-y^2} \end{aligned}}$

10. AP QUESTION - No Calculator - Find the area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$.

VERTICAL CROSS-SECTION	HORIZONTAL CROSS-SECTION
 $A = \int_1^3 (3x^2 + 2x)\, dx$ $x^3 + x^2 \Big]_1^3$ $(27 + 9) - (1 + 1)$ $36 - 2$ 34	Can't do! $y = 3x^2 + 2x$ can't be solved for X.

11. AP QUESTION - Calculator - Find the area of the region in the first quadrant enclosed by the y-axis and the graphs of $y = 3\cos x$ and $y = x$.

VERTICAL CROSS-SECTION



$$A = \int_0^{1.170} (3\cos x - x)\,dx$$

$$A = 2.078$$

HORIZONTAL CROSS-SECTION

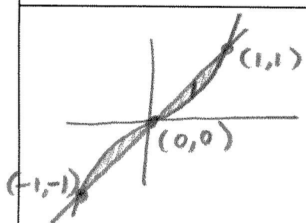
$$\boxed{\begin{aligned} x &= \cos^{-1}\left(\tfrac{1}{3}y\right) \\ x &= y \end{aligned}}$$

$$A = \int_0^{1.170} (y)\,dy + \int_{1.170}^{3} \cos^{-1}\left(\tfrac{1}{3}y\right) dy$$

$$A = 2.078$$

12. AP QUESTION - No Calculator - Find the total area enclosed by the curves $y = x^3$ and $y = x$.

VERTICAL CROSS-SECTION

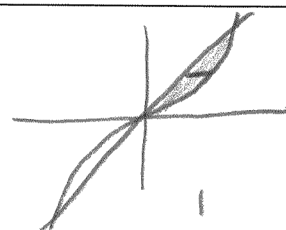


$$A = \int_0^{1} (x - x^3)\,dx$$

$$A = 1/4$$

$$\text{Total area} = 2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}$$

HORIZONTAL CROSS-SECTION



$$\boxed{\begin{aligned} x &= y \\ x &= \sqrt[3]{y} \end{aligned}}$$

$$A = \int_0^{1} (\sqrt[3]{y} - y)\,dy$$

$$A = 1/4$$

$$\text{Total area} = 2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}$$