

Set 8: Multiple-Choice Questions on Further Applications of Integration

The aim of these questions is mainly to reinforce how to set up definite integrals, rather than how to integrate or evaluate them. Therefore we encourage using a graphing calculator wherever helpful.

1. A particle moves along a line in such a way that its position at time t is given by $s = t^3 - 6t^2 + 9t + 3$. Its direction of motion changes when
 - (A) $t = 1$ only
 - (B) $t = 2$ only
 - (C) $t = 3$ only
 - (D) $t = 1$ and $t = 3$
 - (E) $t = 1, 2$, and 3
2. A body moves along a straight line so that its velocity v at time t is given by $v = 4t^3 + 3t^2 + 5$. The distance the body covers from $t = 0$ to $t = 2$ equals
 - (A) 34
 - (B) 55
 - (C) 24
 - (D) 44
 - (E) none of these
3. A particle moves along a line with velocity $v = 3t^2 - 6t$. The total distance traveled from $t = 0$ to $t = 3$ equals
 - (A) 9
 - (B) 4
 - (C) 2
 - (D) 16
 - (E) none of these
4. The net change in the position of the particle in Question 3 is
 - (A) 2
 - (B) 4
 - (C) 9
 - (D) 16
 - (E) none of these
5. The acceleration of a particle moving on a straight line is given by $a = \cos t$, and when $t = 0$ the particle is at rest. The distance it covers from $t = 0$ to $t = 2$ is
 - (A) $\sin 2$
 - (B) $1 - \cos 2$
 - (C) $\cos 2$
 - (D) $\sin 2 - 1$
 - (E) $-\cos 2$
6. During the worst 4-hr period of a hurricane the wind velocity, in miles per hour, is given by $v(t) = 5t - t^2 + 100$, $0 \leq t \leq 4$. The average wind velocity during this period (in mph) is
 - (A) 10
 - (B) 100
 - (C) $102\frac{1}{2}$
 - (D) $104\frac{2}{3}$
 - (E) $108\frac{2}{3}$
7. A car accelerates from 0 to 60 mph in 10 sec, with constant acceleration. (Note that 60 mph = 88 ft/sec.) The acceleration (in ft/sec²) is
 - (A) 5.3
 - (B) 6
 - (C) 8
 - (D) 8.8
 - (E) none of these

For Questions 8–10 use the following information: The velocity \mathbf{v} of a particle moving on a curve is given, at time t , by $\mathbf{v} = t\mathbf{i} - (1-t)\mathbf{j}$. When $t = 0$, the particle is at point $(0,1)$.

*8. At time t the position vector \mathbf{R} is

- (A) $\frac{t^2}{2}\mathbf{i} - \frac{(1-t^2)}{2}\mathbf{j}$ (B) $\frac{t^2}{2}\mathbf{i} + \frac{(1-t)^2}{2}\mathbf{j}$
 (C) $\frac{t^2}{2}\mathbf{i} + \frac{t^2 - 2t}{2}\mathbf{j}$ (D) $\frac{t^2}{2}\mathbf{i} + \frac{t^2 - 2t + 2}{2}\mathbf{j}$
 (E) $\frac{t^2}{2}\mathbf{i} + (1-t)^2\mathbf{j}$

*9. The acceleration vector at time $t = 2$ is

- (A) $\mathbf{i} + \mathbf{j}$ (B) $\mathbf{i} - \mathbf{j}$ (C) $\mathbf{i} + 2\mathbf{j}$ (D) $2\mathbf{i} - \mathbf{j}$ (E) none of these

*10. The speed of the particle is at a minimum when t equals

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 1.5 (E) 2

*11. A particle moves along a curve in such a way that its position vector and velocity vector are perpendicular at all times. If the particle passes through the point $(4, 3)$, then the equation of the curve is

- (A) $x^2 + y^2 = 5$ (B) $x^2 + y^2 = 25$ (C) $x^2 + 2y^2 = 34$
 (D) $x^2 - y^2 = 7$ (E) $2x^2 - y^2 = 23$

*12. The acceleration of a particle is given by the vector $\mathbf{a} = e^{-t}\mathbf{i} + e^t\mathbf{j}$. When $t = 0$, the particle has velocity $\mathbf{v} = 2\mathbf{i}$. Then the velocity vector at any time t is

- (A) $(3 - e^{-t})\mathbf{i}$ (B) $(1 - e^{-t})\mathbf{i} + (e^t - 1)\mathbf{j}$ (C) $(2 - e^{-t})\mathbf{i} + e^t\mathbf{j}$
 (D) $(2 - e^{-t})\mathbf{i} + (e^t - 1)\mathbf{j}$ (E) $(3 - e^{-t})\mathbf{i} + (e^t - 1)\mathbf{j}$

*13. The velocity of a particle is given by $(3 - e^{-t})\mathbf{i} + (e^t - 1)\mathbf{j}$. When $t = 0$, the particle is at $(1,2)$. The position vector \mathbf{R} at time t is

- (A) $(3t + e^{-t})\mathbf{i} + (e^t - t)\mathbf{j}$ (B) $(3t + e^{-t} + 1)\mathbf{i} + (e^t - t + 1)\mathbf{j}$
 (C) $(3t + e^{-t})\mathbf{i} + (e^t - t + 1)\mathbf{j}$ (D) $(3t + e^{-t} + 1)\mathbf{i} + (e^t - t + 2)\mathbf{j}$
 (E) none of these

14. Suppose the current world population is 6 billion and the population t yr from now is estimated to be $P(t) = 6e^{0.024t}$. On the basis of this supposition, the average population of the world, in billions, over the next 25 yr will be approximately

- (A) 6.75 (B) 7.2 (C) 7.8 (D) 8.2 (E) 9.0

15. If a quantity $Q(t)$ is growing at the rate of 5% per year and Q now equals Q_0 , then in t yr Q will equal
 (A) $Q_0(1.05t)$ (B) $Q_0(1.05)^t$ (C) $Q_0e^{0.05t}$
 (D) $Q_0e^{0.045t}$ (E) none of these
16. A growth rate of 3% per year is equivalent to a continuous growth rate of
 (A) 3% (B) 2.99% (C) 2.98% (D) 2.97% (E) 2.96%
17. A stone is thrown upward from the ground with an initial velocity of 96 ft/sec. Its average velocity (given that $a(t) = -32$ ft/sec²) during the first 2 sec is
 (A) 16 ft/sec (B) 32 ft/sec (C) 64 ft/sec
 (D) 80 ft/sec (E) 96 ft/sec
18. Suppose the amount of a drug in a patient's bloodstream t hr after intravenous administration is $30/(t + 1)^2$ mg. The average amount in the bloodstream during the first 4 hr is
 (A) 6.0 mg (B) 11.0 mg (C) 16.6 mg
 (D) 24.0 mg (E) none of these
19. A rumor spreads through a town at the rate of $(t^2 + 10t)$ new people per day. Approximately how many people hear the rumor during the second week after it was first heard?
 (A) 1535 (B) 1894 (C) 2000
 (D) 2219 (E) none of these
20. Oil is leaking from a tanker at the rate of $1000e^{-0.3t}$ gal/hr, where t is given in hours. A general Riemann sum for the amount of oil that leaks out in the next 8 hr, where the interval $[0, 8]$ has been partitioned into n subintervals, is
 (A) $\sum_{k=1}^n e^{-0.3t_k} \Delta t$ (B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n e^{-0.3t_k} \Delta t$
 (C) $\lim_{n \rightarrow \infty} \sum_{k=1}^n 1000e^{-0.3t_k} \Delta t$ (D) $\sum_{k=1}^n 1000e^{-0.3t_k} \Delta t$
 (E) $1000 \sum_{k=1}^n e^{0.3t_k} \Delta t$
21. In Question 20, the total number of gallons of oil that will leak out during the next 8 hr is approximately
 (A) 1271 (B) 3031 (C) 3161 (D) 4323 (E) 11,023

22. Assume that the density of vehicles (number per mile) during morning rush hour, for the 20-mi stretch along the New York State Thruway southbound from the Tappan Zee Bridge, is given by $f(x)$, where x is the distance, in miles, south of the bridge. Which of the following gives the number of vehicles (on this 20-mi stretch) from the bridge to a point x mi south of the bridge?

- (A) $\int_0^x f(t) dt$ (B) $\int_x^{20} f(t) dt$ (C) $\int_0^{20} f(x) dx$
- (D) $\sum_{k=1}^n f(x_k) \Delta x$ (where the 20-mi stretch has been partitioned into n equal subintervals)
- (E) none of these

23. The center of a city that we will assume is circular is on a straight highway. The radius of the city is 3 mi. The density of the population, in thousands of people per square mile, is given approximately by $f(r) = 12 - 2r$ at a distance r mi from the highway. The population of the city is given by the integral

- (A) $\int_0^3 (12 - 2r) dr$ (B) $2 \int_0^3 (12 - 2r) \sqrt{9 - r^2} dr$
- (C) $4 \int_0^3 (12 - 2r) \sqrt{9 - r^2} dr$ (D) $\int_0^3 2\pi r(12 - 2r) dr$
- (E) $2 \int_0^3 2\pi r(12 - 2r) dr$

24. The population density of Winnipeg, which is located in the middle of the Canadian prairie, drops dramatically as distance from the center of town increases. This is shown in the following table:

x = distance (in mi) from the center	0	2	4	6	8	10
$f(x)$ = density (hundreds of people/mi ²)	50	45	40	30	15	5

Using a Riemann sum, we can calculate the population living within a 10-mi radius of the center to be approximately

- (A) 608,500 (B) 650,000 (C) 691,200
- (D) 702,000 (E) 850,000
25. If a factory continuously dumps pollutants into a river at the rate of $\frac{\sqrt{t}}{180}$ tons per day, then the amount dumped after 7 weeks is approximately
- (A) 0.07 ton (B) 0.90 ton (C) 1.55 tons
- (D) 1.9 tons (E) 1.27 tons

26. A roast at 160°F is put into a refrigerator whose temperature is 45°F . The temperature of the roast is cooling at time t at the rate of $(-9e^{-0.08t})^{\circ}\text{F}$ per minute. The temperature, to the nearest degree F, of the roast 20 min after it is put in the refrigerator is
 (A) 45° (B) 70° (C) 81° (D) 90° (E) 115°
27. How long will it take to release 9 tons of pollutant if the rate at which pollutant is being released is $te^{-0.3t}$ tons per week?
 (A) 10.2 weeks (B) 11.0 weeks (C) 12.1 weeks
 (D) 12.9 weeks (E) none of these
28. If you deposit \$1000 today at 8% interest compounded continuously, it will grow at the rate of $80e^{0.08t}$ dollars per year. In 6 years it will be worth (in dollars)
 (A) 616.07 (B) 1129.29 (C) 1292.86
 (D) 1616.07 (E) 6160.74
29. The average area, in square inches, of all circles with radii between 2 and 5 in. is
 (A) 7π (B) 11π (C) 13π (D) $\frac{29\pi}{2}$ (E) 17π
30. What is the exact total area bounded by the curve $f(x) = x^3 - 4x^2 + 3x$ and the x -axis?
 (A) -2.25 (B) 2.25 (C) 3 (D) 3.083 (E) none of these
31. Water is leaking from a tank at the rate of $(-0.1t^2 - 0.3t + 2)$ gal/hr. The total amount, in gallons, that will leak out in the next 3 hr is approximately
 (A) 1.00 (B) 2.08 (C) 3.13 (D) 3.48 (E) 3.75
32. A bacterial culture is growing at the rate of $1000e^{0.03t}$ bacteria in t hr. The total increase in bacterial population during the second hour is approximately
 (A) 46 (B) 956 (C) 1046 (D) 1061 (E) 2046
33. An 18-wheeler traveling at speed v mph gets about $(4 + 0.01v)$ mpg (miles per gallon) of diesel fuel. If its speed is $80\frac{t+1}{t+2}$ mph at time t , then the amount, in gallons, of diesel fuel used during the first 2 hr is approximately
 (A) 20 (B) 21.5 (C) 23.1 (D) 24 (E) 25