## Question 3

| Distance $x$ (cm) | 0 | 1 | 5 | 6 | 8 |
|---|---|---|---|---|---|
| Temperature $T(x)$ (°C) | 100 | 93 | 70 | 62 | 55 |

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire $x$ cm from the heated end. The function $T$ is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every $x$ in the interval $0 < x < 8$? Explain your answer.

# AP® CALCULUS AB
# 2005 SCORING GUIDELINES

## Question 3

| | |
|---|---|
| (a) $\dfrac{T(8)-T(6)}{8-6} = \dfrac{55-62}{2} = -\dfrac{7}{2}\,{}^\circ\text{C/cm}$ | 1 : answer |
| (b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$<br>Trapezoidal approximation for $\int_0^8 T(x)\,dx$:<br>$A = \dfrac{100+93}{2}\cdot 1 + \dfrac{93+70}{2}\cdot 4 + \dfrac{70+62}{2}\cdot 1 + \dfrac{62+55}{2}\cdot 2$<br>Average temperature $\approx \dfrac{1}{8}A = 75.6875^\circ\text{C}$ | 3 : 1 : $\dfrac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer |
| (c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45^\circ\text{C}$<br>The temperature drops $45^\circ\text{C}$ from the heated end of the wire to the other end of the wire. | 2 : 1 : value; 1 : meaning |
| (d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70-93}{5-1} = -5.75$.<br>Average rate of change of temperature on $[5, 6]$ is $\dfrac{62-70}{6-5} = -8$.<br>No. By the MVT, $T'(c_1) = -5.75$ for some $c_1$ in the interval $(1, 5)$ and $T'(c_2) = -8$ for some $c_2$ in the interval $(5, 6)$. It follows that $T'$ must decrease somewhere in the interval $(c_1, c_2)$. Therefore $T''$ is not positive for every $x$ in $[0, 8]$. | 2 : 1 : two slopes of secant lines; 1 : answer with explanation |
| Units of $^\circ$C/cm in (a), and $^\circ$C in (b) and (c) | 1 : units in (a), (b), and (c) |

## Question 6

| $t$ (sec) | 0 | 15 | 25 | 30 | 35 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $v(t)$ (ft/sec) | –20 | –30 | –20 | –14 | –10 | 0 | 10 |
| $a(t)$ $\left(\text{ft}/\text{sec}^2\right)$ | 1 | 5 | 2 | 1 | 2 | 4 | 2 |

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity $v$, measured in feet per second, and acceleration $a$, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time $t$ when $v(t) = -5$ ? Justify your answer.

(d) For $0 < t < 60$, must there be a time $t$ when $a(t) = 0$ ? Justify your answer.

## Question 6

| | |
|---|---|
| (a) $\int_{30}^{60} \lvert v(t) \rvert \, dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.<br>Trapezoidal approximation for $\int_{30}^{60} \lvert v(t) \rvert \, dt$ :<br>$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185$ ft | $2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$ |
| (b) $\int_0^{30} a(t) \, dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.<br>$\int_0^{30} a(t) \, dt = \int_0^{30} v'(t) \, dt = v(30) - v(0)$<br>$= -14 - (-20) = 6$ ft/sec | $2 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$ |
| (c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a $t$ in $(35, 50)$ so that $v(t) = -5$. | $2 : \begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$ |
| (d) Yes. Since $v(0) = v(25)$, the MVT guarantees a $t$ in $(0, 25)$ so that $a(t) = v'(t) = 0$. | $2 : \begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$ |
| Units of ft in (a) and ft/sec in (b) | 1 : units in (a) and (b) |

## 2001 SCORING GUIDELINES

### Question 2

| $t$ (days) | $W(t)$ (°C) |
|---|---|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function $W$ of time $t$. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function $P$, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time $t$, where $t$ is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function $P$ defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

# 2001 SCORING GUIDELINES

## Question 2

(a) Difference quotient; e.g.

$$W'(12) \approx \frac{W(15) - W(12)}{15 - 12} = -\frac{1}{3} \text{ °C/day or}$$

$$W'(12) \approx \frac{W(12) - W(9)}{12 - 9} = -\frac{2}{3} \text{ °C/day or}$$

$$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = -\frac{1}{2} \text{ °C/day}$$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer (with units)} \end{cases}$

(b) $\frac{3}{2}(20 + 2(31) + 2(28) + 2(24) + 2(22) + 21) = 376.5$

Average temperature $\approx \frac{1}{15}(376.5) = 25.1\,°\text{C}$

$2 : \begin{cases} 1 : \text{trapezoidal method} \\ 1 : \text{answer} \end{cases}$

(c) $P'(12) = 10e^{-t/3} - \frac{10}{3}te^{-t/3}\Big|_{t=12}$

$= -30e^{-4} = -0.549\text{ °C/day}$

This means that the temperature is decreasing at the rate of 0.549 °C/day when $t = 12$ days.

$2 : \begin{cases} 1 : P'(12) \text{ (with or without units)} \\ 1 : \text{interpretation} \end{cases}$

(d) $\frac{1}{15}\int_0^{15}\left(20 + 10te^{-t/3}\right)dt = 25.757\,°\text{C}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and average value constant} \\ 1 : \text{answer} \end{cases}$