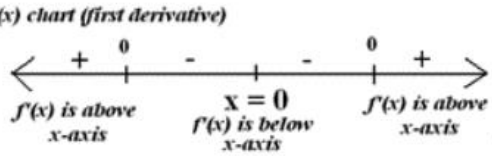
# AP Calculus AB Review Week 3 Solutions

2008

11. 

Therefore, Choice B.

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14. Nothing about $f'(x)$ is given, eliminating A,B and C. E

Only 29% of the students answered this question correctly; common wrong answer is D.
D is not the correct answer because we do not know the sign of $f''(x)$ to the immediate left & right of x = 1.
Just because $f''(x) = 0$ doesn't mean there is an inflection point – the sign of $f''(x)$ must change to be an inflection point.

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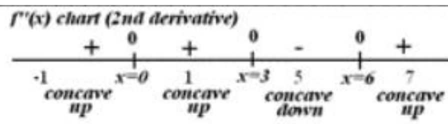
20. Points of inflection at x = 3 and x = 6, 

so Answer: D

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24. $f(2) = 1 \leftrightarrow (2,1)$ and $m = f'(2) = 4$

$y - y_1 = m(x - x_1) \Rightarrow y - 1 = 4(x-2) \rightarrow y = 4x - 7 \Rightarrow y(1.9) = 4(1.9) - 7 = 7.6 - 7 = .6$ B

OR $f(1.9) \approx f(2) + f'(2)(-.1) = 1 + (4)(-.1) = 1 - .4 = .6$

76. $f$ increasing $\Rightarrow f' \geq 0$ One could interpret the answer to be $(-2,1) \cup (1,3)$ instead, but it is NOT listed as a choice. $\therefore$ B [-2,3]

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78. $f$ increasing $\rightarrow f'(x) \geq 0$ Graph: $f'(x) = \sin(x^3 - x) \geq 0$ when curve is above the $x$-axis on [-1,1.691] B

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80. Points of inflection of $f$ occur when $f''$ changes signs. This can happen when $f'$ changes from increasing to decreasing or decreasing to increasing at relative max/min of the graph of $f'$ or when the graph of $f''$ passes through the $x$-axis. When you graph $f'$, there are 5 relative max/min on (–2,2). The graph of $f''$ passes through the $x$-axis 5 times on (–2,2). E

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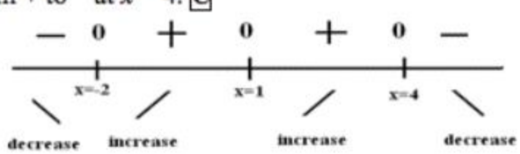
84. $f$ has a rel max when $f'$ changes from + to –. The graph of $f'(x)$ and it's chart show that $f'$ changes from + to – at $x = 4$. C

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88. $S = 4\pi r^2 \rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(-2) = -48\pi$ C

2003

7. When a derivative is below the x-axis it is negative and thus $f(x)$ is decreasing and when a derivative is above the x-axis it is positive and thus $f(x)$ is increasing. From –2 to 0 the derivative is above the x-axis, so $f(x)$ is increasing. B

12. Rate of change of volume is $\frac{dV}{dt}$. When doing a direct proportion, say a is directly proportional to b, you get a = kb. Applying this gives $\frac{dV}{dt} = k\sqrt{V}$ **E**

10. For $f(x)$ to be negative means the entire curve would be under the x-axis thus eliminating E. For $f'(x)$ to be negative would mean the curve is always decreasing thus eliminating A and C. For $f''(x)$ to be negative would mean the curve is always concave down eliminating D **B**

15. If a function is decreasing, then its derivative must be negative or less than zero. Notice that $x^2 - \frac{2}{x} < 0$ is trivially false for 0, and for $x < 0$, both $x^2$ and $-\frac{2}{x}$ are positive making $x^2 - \frac{2}{x} < 0$ false. So consider $x > 0$. Then $x^2 - \frac{2}{x} < 0$ is the same as $x^2 < \frac{2}{x}$ or $x^3 < 2$ or $x < \sqrt[3]{2}$ This makes $(0, \sqrt[3]{2}]$. There is controversy on whether the point where the derivative equals zero should be included in the interval. **D**

17. The graph would be concave down when the second derivative is negative. The first derivative is $2xe^x + 2e^x$ Using this as an aid, the second derivative is $2xe^x + 2e^x + 2e^x$ or $2xe^x + 4e^x$. Factor out $e^x$ giving $e^x(2x + 4)$. As $e^x$ is always positive, disregard it. Find when negative. $2x + 4 < 0 \rightarrow 2x < -4 \rightarrow x < -2$ **A**

18. Since it only has two zeros, the signs of $g'(x)$ in all intervals must not change. A function decreases when its derivative is negative or below the x-axis. From the table, this occurs between – 2 and 2. **A**

21. A function has an inflection point where the second derivative is 0 and its sign changes on opposite sides of this zero point (graphically, the curve has a point on the x-axis and the curve on one side of this zero point is above the x-axis and below the x-axis on the other side). There are 3 zero points. The one at b does not meet the criteria. **A**

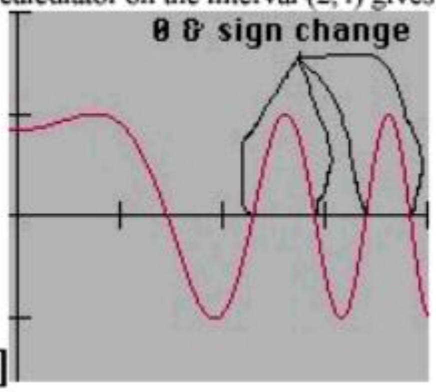
22. If it is a line, then its equation must be y = mx + b where m is the slope, or $f'(x)$. The slope is constant for a line. Using the two points (1,0) and (0,6) then the slope is $\frac{0-6}{1-0} = -6$. The y-intercept of this line is 6 from the graph. Thus the derivative is $-6x + 6$. The function will be the integral of this or $f(x) = -3x^2 + 6x + C$. Putting in $f(0) = 5$ gives $5 = -3(0)^2 + 6(0) + C$ or $C = 5$. Now $f(x) = -3x^2 + 6x + 5$ so $f(1) = 8$. **D**

25. A particle is at rest when its velocity (first derivative of the position) is 0. Taking the first derivative gives $6t^2 - 42t + 72$. Setting this to 0 and then factoring gives $6(t^2 - 7t + 12) = 6(t-4)(t-3) = 0$ so $v(t) = 0$ when $t = 3,4$ **E**

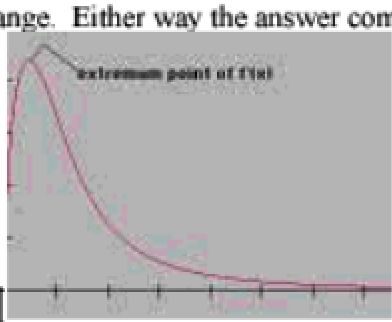
28. The first derivative positive means the function is increasing and the second derivative positive means that the curve is concave up. A concave up increasing curve must have its slope line getting more vertical as x increases, so the rate of change must continue to grow as x increases. The rate of change of g(x) from x = 4 to x = 5 was 6. The change then from x = 5 to x = 6 must be greater than 6. The only value showing this type of change would be g(x) = 27. [E]

76. Acceleration is the derivative of the velocity, so $a(t) = -3.69 \sin(0.9t)$. Putting in t = 4 gives $-(-1.633)$ [C]

81. A curve has a relative extrema when its derivative is 0 and the sign of the derivative changes from one side of the zero to the other. (Graphically, the continuous derivative goes from above the x-axis to below the x-axis or visa versa. Graphing this derivative using calculator on the interval (2,4) gives a curve with 4 zeroes or 4 relative extrema. [D]




87. An inflection point happens where the second derivative is 0 and it changes sign crossing the zero. (Graphically it goes from above the x-axis to below or visa versa). You can graph the first derivative(this scenario is illustrated in the picture below) and find a relative extrema which would make its derivative (the second derivative) 0 and changing sign. (A terrace point would not make it change sign). Then use the calculator to find the x coordinate of this extremum point. You could also do math9(f(X),X,X) in the y= portion of your graphing calculator (TI 83 or similar) and get the graph of the second derivative. Then look for a zero with change. Either way the answer comes out 0.473



[B]

90. Positive first derivative or rate of change or slope of the tangent line means the function is increasing, or the y values are getting larger as the x values increase. This eliminates C, D; E. Negative second derivative means the curve is concave down. When a curve is concave down its tangent line comes closer to horizontal (rate of change or slope decreases) so the rate of change of y values on equal intervals of the curve would be smaller. This eliminates A. [B]

## 2013 AB/BC #1
## (calculator active)

Unprocessed gravel arrives at the rate of $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)\frac{tons}{hour}$ for $0 \le t \le 8$ *hours*

When $t = 0$, there are 500 tons of unprocessed gravel and during the time interval given, the plant processes gravel at a constant rate of 100 $\frac{tons}{hour}$.

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(d) The amount of unprocessed gravel at the plant on $0 \le t \le 8$ will be a maximum at $t = 0$, $t = 8$, or where $A' = G(t) - 100 = 0$. Using the calculator by graphing and finding the zeros of $A'$, this occurs at $t = 4.923480$. Store this as $a$ in the calculator.

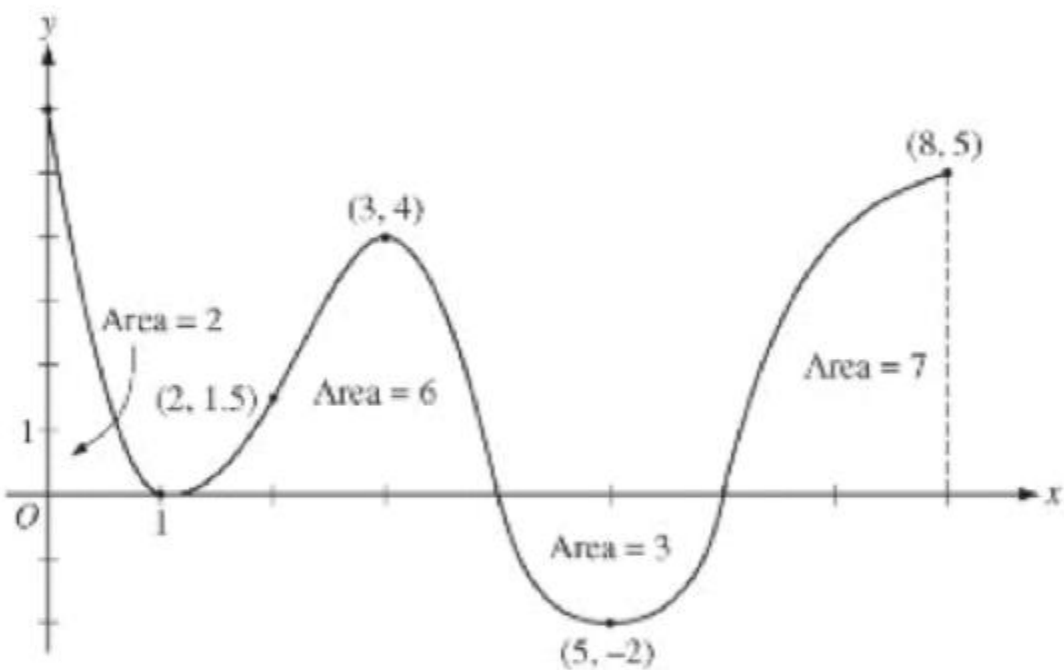
$t = 0,\ A = 500$

$t = 8,\ A = 500 + \int_0^8 G(t)dt - 100(8) = 525.551$

$t = a,\ A = 500 + \int_0^a G(t)dt - 100(a) = 635.376$

More gravel is being **processed** than **arriving** when $t >$ a. The maximum amount of unprocessed gravel at the plant during the workday is $\boxed{635.376\ tons}$

## 2013 AB/BC #4
## (no calculator)




Graph of $f'$

$f$ is a twice-differentiable function and $f(8) = 4$

(a) $f$ has a relative or local minimum at $\boxed{x = 6}$ because $f'$ changes from negative to positive there (it goes from below the $x$-axis to above the $x$-axis at $x = 6$).

(b) $f$ has an absolute minimum on the interval at $x = 0$, $x = 8$, or at the only relative minimum on the interval, $x = 6$. Using the Fundamental Theorem of Calculus,

$$\int_0^8 f'(x)dx = f(8) - f(0) \Rightarrow f(0) = f(8) - \int_0^8 f'(x)dx = 4 - (2 + 6 - 3 + 7) = -8$$

$$\int_6^8 f'(x)dx = f(8) - f(6) \Rightarrow f(6) = f(8) - \int_6^8 f'(x)dx = 4 - (7) = -3$$

$$f(8) = 4$$

So, the absolute minimum of $f$ on $[0,8]$ is $\boxed{-8}$ using the candidates test.

(c) $f$ is concave down when $f'' < 0$ or when $f'$ is decreasing.
$f$ is increasing when $f' > 0$ or when the graph of $f'$ is above the $x$-axis.
Both of these occur on the intervals $\boxed{(0,1) \text{ and } (3,4).}$

(d)

$$g(x) = (f(x))^3 \text{ and } f(3) = -\frac{5}{2} \Rightarrow g'(x) = 3(f(x))^2 f'(x)$$

$$slope = g'(3) = 3(f(3))^2 f'(3) = \boxed{3\left(-\frac{5}{2}\right)^2 (4)} = 75$$

2008 AB #3

Since volume is increasing at 2000 $\frac{cm^3}{\min}$, then $\frac{dV}{dt} = 2000$.

(a) $r = 100$, $h = 0.5$, $\frac{dr}{dt} = 2.5$ and $V = \pi r^2 h$

Taking the derivative with respect to $t$: $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$

Substituting the values: $2000 = \pi(100)^2 \frac{dh}{dt} + (.5)(2\pi)(100)(2.5)$

So, $\frac{dh}{dt} = \boxed{\frac{2000 - 250\pi}{100^2 \pi} \frac{cm}{\min}}$ or $\boxed{0.039 \frac{cm}{\min}}$

(b) $R(t) = 400\sqrt{t} \frac{cm^3}{\min}$

The amount of oil is $V = \int_0^t (2000 - R(t))dt$. The amount of oil is a maximum when $V' = 2000 - R(t) = 2000 - 400\sqrt{t}$ changes from positive to negative.
If you graph $V'(x)$, $V'$ is above the $x$-axis or positive when $x < 25$, $V'(25) = 0$, and $V'$ is below the $x$-axis or negative when $x > 25$. So, the amount of oil is a maximum when
$\boxed{t = 25 \text{ min}}$

**2007 AB 3**

$h(x) = f(g(x)) - 6$ $g$ is increasing, so $g'(x) > 0$

$f$ and $g$ are differentiable, hence continuous

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

Since $h$ is continuous, by the Intermediate Value Theorem, for every $y$-value between –7 and 3, there is at least one value of $r$ between 1 and 3 such that $h(r)$ equals this $y$-value.

Since –5 is between –7 and 3, there is at least one value of $r$ between 1 and 3 such that $h(r) = -5$ .

(b) Since $h$ is continuous and differentiable, by the Mean Value Theorem, there must be a value for $1 < c < 3$ such that $h'(c) = \dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{2} = -5$ .

**2007 AB/BC 5**

(a) The linearization of $r(t)$ is: $r(t) \approx r(a) + r'(a)(t - a)$ .

So, $r(5.4) \approx r(5) + r'(5)(.4) = 30 + 2(.4) = \boxed{30.8\ feet}$

This estimate is $\boxed{\textit{greater than}}$ the true value since $r$ is concave down on the interval $5 < t < 5.4$.

(b) Since $V = \frac{4}{3}\pi r^3$, then $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$

When $t = 5$, $\dfrac{dV}{dt} = 4\pi(30)^2(2) = \boxed{7200\pi \ \dfrac{ft^3}{\min}}$