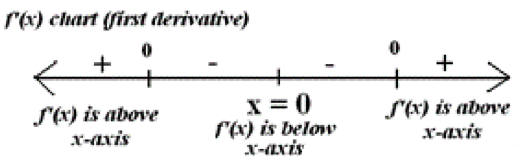
# AP Calculus AB Week 2 Solutions

2008

3. Product Rule → $f'(x) = (x-1)[3(x^2+2)^2(2x)] + (x^2+2)^3(1)$
$= (x^2+2)^2\{6x(x-1) + (x^2+2)\} = (x^2+2)^2(7x^2 - 6x + 2)$ D

8. $f(x) = \cos(3x)$ → $f'(x) = -3\sin(3x)$ Therefore, $f'\left(\frac{\pi}{9}\right) = -3\sin\left(\frac{\pi}{3}\right) = -3\left(\frac{\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2}$ E

11.




Therefore, Choice B.

12. $f'(x) = e^{2/x}(-2x^{-2}) = \frac{-2e^{2/x}}{x^2}$ D

13. Since $f(x) = x^2 + 2x \rightarrow f(\ln x) = (\ln x)^2 + 2\ln x \Rightarrow \frac{d}{dx}(f(\ln x)) = 2(\ln x)\left(\frac{1}{x}\right) + 2\left(\frac{1}{x}\right) = \frac{2\ln x + 2}{x}$ A

18. slope of the tangent line is $y' = 2x + 3 = m$

$x + y = k \rightarrow y = -x + k \Rightarrow m = -1$

Solve $2x + 3 = -1 \rightarrow 2x = -4 \rightarrow x = -2$, $\rightarrow f(-2) = (-2)^2 + 3(-2) + 1 = -1 = y$

$\Rightarrow x + y = k \Rightarrow -2 + (-1) = \boxed{-3}$ A

16. Chain Rule and Implicit Differentiation: $\cos(xy)\left[xy' + y(1)\right] = 1 \Rightarrow (xy')\cos(xy) + y\cos(xy) = 1$

$\frac{dy}{dx} = \boxed{y' = \frac{1 - y\cos(xy)}{x\cos(xy)}}$ Answer: D *Only 37% of students answered it correctly!

21. Velocity increasing $\leftrightarrow v' = x'' > 0$ when $x(t)$ is concave up on $0 < t < 2 \rightarrow$ Answer: A

24. $f(2) = 1 \leftrightarrow (2,1)$ and $m = f'(2) = 4$

$y - y_1 = m(x - x_1) \Rightarrow y - 1 = 4(x-2) \rightarrow y = 4x - 7 \Rightarrow y(1.9) = 4(1.9) - 7 = 7.6 - 7 = .6$ B

OR $f(1.9) \approx f(2) + f'(2)(-.1) = 1 + (4)(-.1) = 1 - .4 = .6$

25. $f$ is differentiable at $x = 2$ → $f'(x) = \begin{cases} c & ,x<2 \\ 2x-c & ,x>2 \end{cases}$ So, $c = 2x - c$ so $f'(2) \Rightarrow 2c = 2(2) \Rightarrow c = 2$

$f$ is continuous at $x = 2$ (Diff→Cont)

$\therefore \lim\limits_{x\to 2^-}(cx+d) = \lim\limits_{x\to 2^+}(x^2 - cx) \Rightarrow 2c + d = 4 - 2c \Rightarrow d = 4 - 4c = 4 - 4(2) = -4$

$\Rightarrow c + d = 2 + (-4) = -2$ $\boxed{\text{B}}$

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26. $y = \tan^{-1}(4x) \rightarrow \frac{dy}{dx} = \frac{1}{1+(4x)^2} \bullet (4) = \frac{4}{1+16x^2}$ $\left.\frac{dy}{dx}\right|_{x=\frac{1}{4}} = \frac{4}{1+16\left(\frac{1}{4}\right)^2} = \frac{4}{2} = 2$ $\boxed{\text{A}}$

28. Since $f(6) = 3, f^{-1}(3) = g(3) = 6 \Rightarrow g'(x) = \frac{1}{f'(g(x))} \Rightarrow g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(6)} = \frac{1}{-2}$ $\boxed{\text{A}}$

ONLY 14% of students answered this correctly.

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82. $a(3) = v'(3) = 0.055$ $\boxed{\text{B}}$ using $\boxed{\text{nDeriv}}$ on your calculator

2003

1. $2(x^3+1)^1(3x^2) = 6x^2(x^3+1)$ $\boxed{E}$

4. $\frac{(3x+2)(2)-(2x+3)(3)}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$ $\boxed{D}$

9. $f'(x) = \frac{1}{x+4+e^{-3x}} \bullet (1-3e^{-3x})$ so $f'(0) = \frac{1}{4+1} \bullet (1-3) = -\frac{2}{5}$ $\boxed{A}$

14. $x^2(2(\cos 2x)) + \sin 2x(2x)$ Factor out 2x giving $2x(x \cos 2x + \sin 2x) =$
$2x(\sin 2x + x \cos 2x)$ $\boxed{E}$

16. The derivative at the point is the slope of the tangent line at the point. Calculate the slope using the two-point formula $\frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$ $\boxed{C}$

22. If it is a line, then its equation must be y = mx + b where m is the slope, or $f'(x)$. The slope is constant for a line. Using the two points (1,0) and (0,6) then the slope is $\frac{0-6}{1-0} = -6$. The y-intercept of this line is 6 from the graph. Thus the derivative is $-6x+6$. The function will be the integral of this or $f(x) = -3x^2 + 6x + C$. Putting in $f(0) = 5$ gives $5 = -3(0)^2 + 6(0) + C$ or $C = 5$. Now $f(x) = -3x^2 + 6x + 5$ so $f(1) = 8$. $\boxed{D}$

24. To get the equation of the tangent line you need a slope (derivative of the curve at the point) and a point. The derivative $f'(x) = 12x^2 - 5$ so the slope at x = –1 will be $f'(-1) = 7$. Using the original equation to find $f(-1)$ is 4 gives the point. The equation of the line is then $(y - 4) = 7(x - -1)$ or $y = 7x + 11$. $\boxed{C}$

25. A particle is at rest when its velocity (first derivative of the position) is 0. Taking the first derivative gives $6t^2 - 42t + 72$. Setting this to 0 and then factoring gives $6(t^2 - 7t + 12) = 6(t - 4)(t - 3) = 0$ so $v(t) = 0$ when $t = 3,4$ $\boxed{E}$

26. Take the derivative implicitly. $6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} - 2y$. Solve for $\frac{dy}{dx}$.

$\frac{dy}{dx}(6y + 2x) = 4x - 2y \rightarrow \frac{dy}{dx} = \frac{4x - 2y}{6y + 2x}$ Substituting in value for x and y will give $\frac{dy}{dx} = \frac{4(3) - 2(2)}{6(2) + 2(3)} = \frac{12 - 4}{12 + 6} = \frac{8}{18} = \frac{4}{9}$ $\boxed{B}$

27. $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ or $f'(x) = \frac{1}{f^{-1}{}'(y)}$ or $f'(x) = \frac{1}{g'(y)}$. Thus since g(2) = 1, then we can say that $f'(1) = \frac{1}{g'(2)}$ or $g'(2) = \frac{1}{f'(1)}$. $f'(x) = 3x^2 + 1$. $f'(1) = 4$ So $g'(2) = \frac{1}{4}$. $\boxed{B}$

76. Acceleration is the derivative of the velocity, so $a(t) = -3.69\sin(0.9t)$. Putting in t = 4 gives $-(-1.633)$ $\boxed{C}$

89. $g'(x) = x f'(x) + f(x)$ by taking the derivative of $x f(x)$. So $g'(2)$, the derivative of g(x) and consequently the slope of the tangent line at any point x would be equal to $2(-5) + 3 = -7$. The only line given that has slope –7 is $\boxed{D}$.

2013 free response

**2013 AB/BC #1**
**(calculator active)**

Unprocessed gravel arrives at the rate of $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)\frac{tons}{hour}$ for $0 \le t \le 8$ *hours*

When $t = 0$, there are 500 tons of unprocessed gravel and during the time interval given, the plant processes gravel at a constant rate of 100 $\frac{tons}{hour}$.

(a) $G'(5) = \boxed{-24.588}\ \frac{tons}{hour^2}$ using the calculator.

This means that at t = 5 hours (5 hours into the workday), the rate at which unprocessed gravel arrives at the plant is decreasing at 24.588 $\frac{tons}{hour^2}$.

**2013 AB #2**
**(calculator active)**

Velocity, $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$ for $0 \le t \le 5$. Position is $s(t)$ and $s(0) = 10$.

(a) *speed* $= |v(t)|$. To find $t$ when speed is 2, solve the following with your calculator: $|v(t)| = 2$, or $|v(t)| - 2 = 0$. Probably best to graph $y = |v(t)| - 2$ on the calculator and find the zeros between 2 and 4 which are at $\boxed{t = 3.128 \text{ and } t = 3.473}$.

**2013 AB/BC #3**
**(no calculator)**

| $t$ (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $C(t)$ (ounces) | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

Water is dripping into a cup for $t$ minutes where $0 \le t \le 6$.
The amount of coffee in the cup is given by a differentiable function $C$.

(a) $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \boxed{12.8 - 11.2\ \frac{\text{ounces}}{\text{min}}}$ or $1.6\frac{\text{ounces}}{\text{min}}$

**2013 AB #6**
**(no calculator)**

$$\frac{dy}{dx} = e^y\left(3x^2 - 6x\right) \quad \text{and} \quad \text{curve passes through } (1,0) \text{ or } f(1) = 0.$$

(a)

$$slope_{(1,0)} = \frac{dy}{dx}_{(1,0)} = e^0(3-6) = -3 \Rightarrow \text{Tangent line} \Rightarrow \boxed{y - 0 = -3(x-1)}$$

$$f(1.2) \approx \boxed{0 - 3(1.2-1)} = -0.6$$

2012 AB/BC #1

| $t$ (minutes) | 0 | 4 | 9 | 15 | 20 |
|---|---|---|---|---|---|
| $W(t)$ (degrees Fahrenheit) | 55.0 | 57.1 | 61.8 | 67.9 | 71.0 |

$W$ is strictly increasing, twice differentiable, and $W(0) = 55$

a. $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \boxed{\dfrac{67.9 - 61.8}{15 - 9}}$ or 1.016666667 or $\boxed{1.017}$

This is the approximate rate that the temperature is increasing, in degrees Fahrenheit/minute, at $t = 12$ minutes.

2012 AB #4

$$f(x) = \sqrt{25 - x^2} \text{ for } -5 \le x \le 5$$

a. $f'(x) = \boxed{\frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x)}$ or $\boxed{\frac{-x}{\sqrt{25 - x^2}}}$

b. The equation of the tangent line at $x = -3$:

$f(-3) = \sqrt{25 - 9} = 4$ and $f'(-3) = \frac{3}{4} \Rightarrow \boxed{y - 4 = \frac{3}{4}(x + 3)}$

c. $g(x) = \begin{cases} \sqrt{25 - x^2} & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5 \end{cases}$

Three conditions for $g$ to be continuous at $x = -3$:

1. $g(-3) = f(-3) = 4$
2. $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} \sqrt{25 - x^2} = 4$ and $\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4 \Rightarrow \lim_{x \to -3} g(x) = 4$
3. $g(-3) = \lim_{x \to -3} g(x)$

Hence, $g$ is continuous at $x = -3$.

2012 AB/BC #5

$$\frac{dB}{dt} = \frac{1}{5}(100 - B) \text{ and } B(0) = 20$$

a. $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = 12$ and $\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = 6$

Hence the bird is gaining weight faster when it weighs $\boxed{\text{40 grams}}$.

2012 AB #6

$$v(t) = \cos\left(\frac{\pi}{6}t\right) \text{ for } 0 \le t \le 12 \text{ and } x(0) = -2$$

a. $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow \frac{\pi}{6}t = \frac{\pi}{2} \Rightarrow t = 3$ and $\frac{\pi}{6}t = \frac{3\pi}{2} \Rightarrow t = 9$

Using a chart for v(t), $v(t) < 0$ when $\boxed{3 < t < 9}$, hence the particle moves to the left on this interval. Note: $3 \le t \le 9$ is accepted also.

2011 AB #1

On $[0,6]$, $v(t) = 2\sin\left(e^{t/4}\right) + 1$, $a(t) = v'(t) = \frac{1}{2}\left(e^{t/4}\right)\cos\left(e^{t/4}\right)$, $x(0) = 2$

a. $v(5.5) = -0.45337$ and $a(5.5) = -1.3585$

The speed of the particle is $\boxed{\text{increasing}}$ at $t = 5.5$ because both $v(5.5)$ and $a(5.5)$ have the same sign.

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2011 AB/BC #2

| $t$ (minutes) | 0 | 2 | 5 | 9 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

a. $H'(3.5) = \frac{H(5) - H(2)}{5 - 2} = \frac{52 - 60}{3} = \boxed{-\frac{8}{3}\ \frac{°\text{Celsius}}{\text{min}}}$

2011 AB/BC #5

a. The line tangent to W is defined by the point (0,W(0)) = (0,1400) and slope

$$\frac{dW}{dt}(0) = \frac{1}{25}(1400 - 300) = 44.$$

So the tangent is $W - 1400 = 44(t - 0) \rightarrow W = 44t + 1400.$

Thus, at $t = \frac{1}{4} \rightarrow W \sim 44(.25) + 1400 = \boxed{1411 \text{ tons}}$

2011 AB #6

$$f(x) = \begin{cases} 1 - 2\sin x & x \le 0 \\ e^{-4x} & x > 0 \end{cases}$$

b. $f'(x) = \begin{cases} -2\cos x & x < 0 \\ -4e^{-4x} & x > 0 \end{cases}$

$-2\cos x = -3 \rightarrow \cos x = \frac{3}{2}$ This can not happen.

$-4e^{-4x} = -3 \rightarrow e^{-4x} = \frac{3}{4} \rightarrow -4x = \ln\left(\frac{3}{4}\right) \rightarrow \boxed{x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)}$