AP Calculus AB Week 4 Solutions

2008

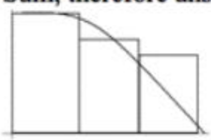
2. $\int x^{-2}dx = \frac{x^{-1}}{-1} + C = -x^{-1} + C$ $\boxed{D}$

4. $\int \sin 2x dx + \int \cos 2x dx \;=\; -\frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x + C$ $\boxed{B}$

9. $g'(x) = f(x)$ So g has a relative maximum when $g'(x)$ or $f(x)$ changes from + to – .
This happens at $x = 1$. $\boxed{D}$

10. Because $f(x)$ is decreasing, Right Riemann Sum $< \int_1^3 f(x)dx <$ Left Riemann Sum

Since decreasing and concave down, Right Riemann Sum < both Midpoint Riemann Sum and Trapezoidal Sum, therefore answer is $\boxed{C.}$



15. $\int \frac{x}{x^2-4}dx \to$ Let $u = x^2 - 4$, $\frac{du}{dx} = 2x \to \frac{du}{2} = xdx \Rightarrow \int \frac{x}{x^2-4}dx \to \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^2-4|+C}$

OR $\int \frac{x}{x^2-4}dx = \square \ln|x^2-4| + C \Rightarrow$ take deriv and compare: $\frac{\square 2x}{x^2-4} \Rightarrow 2\square = 1$ so $\square = \frac{1}{2} \Rightarrow$

$\int \frac{x}{x^2-4}dx = \frac{1}{2}\ln|x^2-4| + C$ Answer: $\boxed{C}$

17. By the Second Fundamental Theorem of Calculus, $g'(x) = f(x)$ and so $g''(x) = f'(x)$
g has a point of inflection where $g'' = f'$ changes sign. This happens when $x = 2$ and $x = 5$ $\boxed{C}$

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79. $\int_{-5}^{5} f(x)dx = \int_{-5}^{2} f(x)dx + \int_{2}^{5} f(x)dx = \int_{-5}^{2} f(x)dx - \int_{5}^{2} f(x)dx = -17 - (-4) = -13$ $\boxed{B}$

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81. Since G(x) is antiderivative for $f(x)$, $\int_2^4 f(x)dx \underset{f.t.c.}{=} G(4) - \underbrace{G(2)}_{-7} = G(4) - (-7) \Rightarrow G(4) = -7 + \int_2^4 f(x)dx$ $\boxed{E}$

83. Graph $f(x) = x^3 - 8x^2 + 18x - 5$ and $g(x) = x + 5$ They intersect at x=1,2,5

Area $= \int_1^2 (f-g)dx + \int_2^5 (g-f)dx = 11.833$ $\boxed{B}$ Or $Area = \int_1^5 |f-g|dx = 11.833$

91. $f_{avg} = \dfrac{1}{b-a}\int_a^b f(x)\,dx = \dfrac{1}{3-(-1)}\int_{-1}^{3}\dfrac{\cos x}{x^2+x+2}dx = 0.183$ [C]

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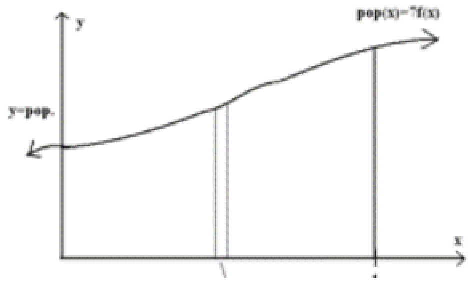
92. The population is given as the sum (accumulation) of the population times the population density on the interval from 0 miles to 4 miles:

$$Population = \int \frac{Pop}{mi^2}\bullet mi^2 = \int_{x=0}^{4}\frac{Pop}{mi^2}\bullet mi\bullet dx = \int_0^4 7\bullet f(x)\,dx$$ [B]



2003

2. u = –4x du = – 4 dx Thus integral is $\dfrac{e^{-4x}}{-4}$ so this give $\dfrac{e^{-4}}{-4} - \dfrac{1}{-4} = \dfrac{1}{4} - \dfrac{e^{-4}}{4}$ [D]

5. Integral of sin x is – cos x so $-\cos\left(\dfrac{\pi}{4}\right) - -\cos(0) = -\dfrac{\sqrt{2}}{2} - -1 = -\dfrac{\sqrt{2}}{2} + 1$ [D]

8. u = x^3 du = $3x^2$dx So $\dfrac{du}{3} = x^2dx$ The integral of $\int\left(\dfrac{\cos u}{3}\right)du = \dfrac{1}{3}\sin u + C$. So substituting in for u gives $\dfrac{1}{3}\sin x^3 + C$ [B]

11. du = 2 dx or dx = $\dfrac{1}{2}$du For limits of integration, if x = 2, then u = 2(x) + 1 = 5 and similarly when x = 0, u = 1. Doing all of the substitutions yields $\dfrac{1}{2}\int_1^5 \sqrt{u}\,du$ [C]

23. This uses what is frequently listed as the Second Fundamental Theorem of Calculus. The derivative of an integral puts you back where you started when one of the limits is a variable expression and the other is a constant with the changes that the variable becomes the variable expression and you multiply everything by the derivative of the limit which is the variable expression (note: If the variable expression is not the upper limit, multiply your answer by a negative 1 to reverse the limit). So the answer will be $2x(\sin(x^2)^3)$ or $2x\sin(x^6)$ [E]

77. Area under a curve is the definite integral of the curve. The area under the curve f $f(x)$ from (–3,3) would be the sum of the given areas, with those under the x-axis as negative areas. Thus $\int_{-3}^{3}(f(x))\,dx = -2$. However,

$$\int_{-3}^{3}(f(x)+1)\,dx = \int_{-3}^{3}(f(x))\,dx + \int_{-3}^{3}(1)\,dx = \int_{-3}^{3}(f(x))\,dx + 6 = -2 + 6 = 4 \quad \boxed{C}$$

82. The change of altitude would be the integral of r(t) on the interval where it is negative. Graph r(t) on calculator and note those intervals that the graph is below the x-axis (negative). These two points are 1.572 and 3.514. $\boxed{A}$

85. The right sum will over approximate a increasing curve and under approximate a decreasing curve. This means either A or C. On a concave up curve, the trapezoids will be above the curve. On a concave down they will be below the curve and consequently under approximate it. This makes the answer $\boxed{A}$

88. This question is asking you to find the function whose <u>average value</u> is 1. D and E have average value of 2. A has more values higher than 1 than less than 1. B has more values less than 1 than higher than 1. Thus, C, where linearly there are an equal number of values above 1 as below is the answer. $\boxed{C}$

92. g will be decreasing if its derivative (slope) is less than 0. This uses what is frequently listed as the Second Fundamental Theorem of Calculus. The derivative of an integral puts you back where you started when one of the limits is a variable expression and the other is a constant with the changes that the variable becomes the variable expression and you multiply everything by the derivative of the limit which is the variable expression (note: If the variable expression is not the upper limit, multiply your answer by a negative 1 to reverse the limit). Thus, g ' (x) will be $\sin(x^2)$. Graph this on [–1,3] using your calculator to see where it is less than 0 (graphically below the x-axis). The graph will be below the x-axis on the interval $1.772 \le x \le 2.507$. $\boxed{D}$ There is controversy on whether the points where the derivative equals zero should be included in the interval.

2013 AB/BC #1
(calculator active)

Unprocessed gravel arrives at the rate of $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right) \frac{tons}{hour}$ for $0 \le t \le 8$ *hours*

When $t = 0$, there are 500 tons of unprocessed gravel and during the time interval given, the plant processes gravel at a constant rate of 100 $\frac{tons}{hour}$.

(b) The amount of unprocessed gravel that arrives to the plant during the hours given is

$$\int_0^8 G(t)\,dt = \boxed{825.551 \text{ tons}}$$

2013 AB/BC #3
(no calculator)

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Water is dripping into a cup for t minutes where $0 \le t \le 6$.
The amount of coffee in the cup is given by a differentiable function C.

(c) Using a midpoint sum with three intervals to approximate the integral,

$$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}\left[(2-0)C(1) + (4-2)C(3) + (6-4)C(5)\right]$$

$$= \boxed{\frac{1}{6}\left[2(5.3) + 2(11.2) + 2(13.8)\right]} = 10.1$$

The average amount of coffee, in ounces, in the cup from $t = 0$ to $t = 6$ minutes.

2011 AB/BC #2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

b. $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature, in degrees Celsius, of the tea from $0 \le t \le 10$ minutes.

The Trapezoidal Approximation:

$$\frac{1}{10}\int_0^{10} H(t)\,dt \sim$$

$$\frac{1}{10}\left[\frac{1}{2}(2)(66+60)+\frac{1}{2}(3)(60+52)+\frac{1}{2}(4)(52+44)+\frac{1}{2}(1)(44+43)\right] = \boxed{52.950^\circ \mathit{Celsius}}$$

c. $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = \boxed{-23^\circ \mathit{Celsius}}$

This is the change, in degrees Celsius, of the temperature of the tea for $0 \le t \le 10$ minutes. This indicates the teas has cooled by $23^\circ \mathit{Celsius}$ in that time interval (or that the temperature of the tea changed by $-23^\circ \mathit{Celsius}$.)

AB/BC 1

Rate that snow accumulates: $f(t)=7te^{\cos t}\ \frac{ft^3}{hr}$

Rate that snow removed: $g(t)=\begin{cases}0 & \text{for } 0\le t<6\\ 125 & \text{for } 6\le t<7\\ 108 & \text{for } 7\le t\le 9\end{cases}\ \frac{ft^3}{hr}$

(a) Snow that has accumulated by 6am: $\int_0^6 f(t)\,dt = \boxed{142.275\ ft^3}$

(b) Rate of change of the volume of snow on the driveway at 8am:

$$f(8)-g(8)=48.41703221-108=\boxed{-59.583\ \frac{ft^3}{hr}}$$

(c) $h(t)$ is the total amount of snow removed when $0\le t\le 9$:

$$h(t)=\begin{cases}0 & \text{for } 0\le t\le 6\\ 0+\int_6^t 125\,dx & \text{for } 6<t\le 7\\ 125+\int_7^t 108\,dx & \text{for } 7<t\le 9\end{cases}$$

OR

$$h(t)=\begin{cases}0 & \text{for } 0\le t\le 6\\ 125(t-6) & \text{for } 6<t\le 7\\ 125+108(t-7) & \text{for } 7<t\le 9\end{cases}$$

Both of the solutions above are correct and each would be accepted for full credit on the AP exam. The top solution uses the fundamental theorem of Calculus and the bottom solution uses linearization.

(d) The amount of snow on the driveway, in ft^3, at 9am will be the amount accumulated from midnight until 9am minus the amount removed from 6am until 9am:

$$\int_0^9 f(t)\,dt-\int_6^7 g(t)\,dt-\int_7^9 g(t)\,dt=\boxed{26.335\ ft^3}$$

OR

$$\int_0^9 f(t)\,dt-h(9)=\boxed{26.335\ ft^3}$$

AB/BC 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx \frac{1}{8}\left[\frac{1}{2}(2)(0+4)+\frac{1}{2}(3)(4+13)+\frac{1}{2}(2)(13+21)+\frac{1}{2}(1)(21+23)\right] =$

$\boxed{10.6875}$ or 10.688

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of entries (in hundreds) in the box from noon until 8pm.