AP Calculus AB

Review Week 4

Techniques of Integration

Advanced Placement AAP Review will be held in **room 315** and **312** on Tuesdays and Thursdays.

The week of April 13th we will be reviewing **Techniques of Integration**

The session will begin in room 315 with a brief review of the weekly topic.

Instruction will be from 3:00 pm to 3:15 pm

Once we have reviewed the topic you may begin practicing the questions in your review packet.

Answers will be posted in room 315 and 312 all week and will be posted on line after 3:00 pm on Friday the week of review.

If you have difficulty with a question look at the detailed answer postings BEFORE you ask your teacher for help.

Get a hint….**DON'T COPY THE ANSWER!!! THAT IS NOT HELPFUL!!**

When you have completed a question…**REFLECT!!!!** Ask yourself what skill you used to solve that problem and write that down!!

Once we have completed the weekly review, keep it to study from as we get closer to the exam.

Techniques of Integration

Brief Review

Riemann Sums…Estimation of area under the curve…you should be able to do left, right and midpoint...use RECTANGLES…usually involves a table.

Trapezoidal Approximation…same as Riemann's but use trapezoids.

Integrals are AREA under the curve…DISTANCE.

Indefinite integrals are evaluated using ANTIDIFFERENTIATION…don't forget C…you can find C if they give you a point on the original curve.

Definite integrals can be evaluated using the second part of the fundemental theorem, geometry, your calculator.

You take the derivative of an integral using the first part of the fundamental theorem.

Total distance is absolute value.

TECHNIQUE:

- U-DU Substitution

THIS IS OUR ONLY TECHNIQUE! If you see a problem that you have to integrate, try u/du substituiton

Integrals are ACCUMULATORS…they add things up.

Acceleration goes backward through an integral to velocity.

Velocity goes backward through an integral to position.

2008 Exam NON Calculator

2. $\int \frac{1}{x^2} dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

4. $\int \left(\sin(2x) + \cos(2x)\right) dx =$

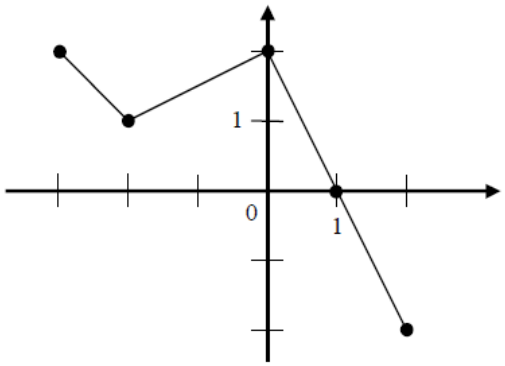
(A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(C) $2\cos(2x) + 2\sin(2x) + C$

(D) $2\cos(2x) - 2\sin(2x) + C$

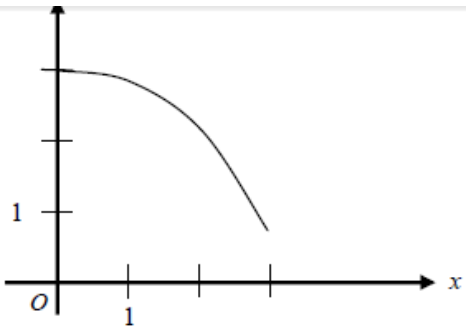
(E) $-2\cos(2x) + 2\sin(2x) + C$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

15. $\int \frac{x}{x^2-4}\,dx =$

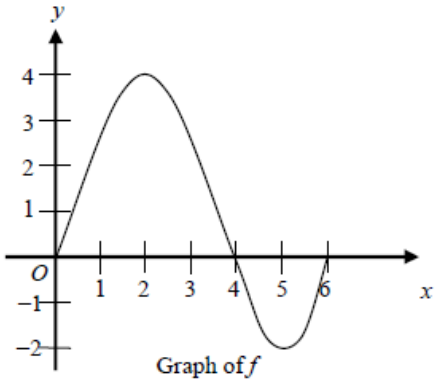
(A) $\frac{-1}{4\left(x^2-4\right)^2}+C$

(B) $\frac{1}{2\left(x^2-4\right)}+C$

(C) $\frac{1}{2}\ln\left|x^2-4\right|+C$

(D) $2\ln\left|x^2-4\right|+C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x=2$ and $x=5$. Let g be the function defined by $g(x)=\int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

2008 EXAM CALCULATOR

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_{2}^{4} f(t)\,dt$

(D) $\int_{2}^{4} (-7 + f(t))\,dt$

(E) $-7 + \int_{2}^{4} f(t)\,dt$

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667

(B) 11.833

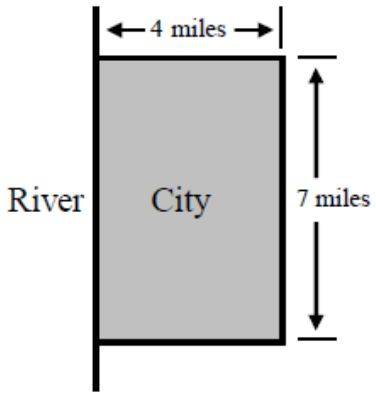
(C) 14.583

(D) 21.333

(E) 32

91. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval [−1, 3]?

(A) −0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)dx$

(B) $7\int_0^4 f(x)dx$

(C) $28\int_0^4 f(x)dx$

(D) $\int_0^7 f(x)dx$

(E) $4\int_0^7 f(x)dx$

2003 Exam Non-Calculator

2. $\int_0^1 e^{-4x}dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4}-1$ (D) $\frac{1}{4}-\frac{e^{-4}}{4}$ (E) $4-4e^{-4}$

5. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2}-1$ (D) $-\frac{\sqrt{2}}{2}+1$ (E) $\frac{\sqrt{2}}{2}-1$

8. $\int x^2\cos\left(x^3\right)dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right)+C$

(B) $\frac{1}{3}\sin\left(x^3\right)+C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right)+C$

(D) $\frac{x^3}{3}\sin\left(x^3\right)+C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

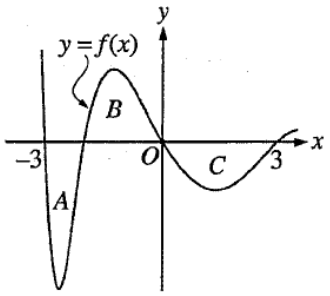
11. Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2}\sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2\sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5\sqrt{u}\,du$ (D) $\int_0^2\sqrt{u}\,du$ (E) $\int_1^5\sqrt{u}\,du$

23. $\frac{d}{dx}\left(\int_0^{x^2}\sin\left(t^3\right)dt\right) =$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

2003 Exam Calculator



77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)dx$?

(A) –2 (B) –1 (C) 4 (D) 7 (E) 12

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

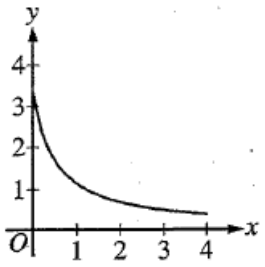
(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

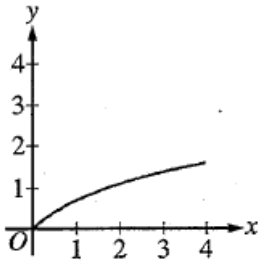
(E) $\int_{0}^{2.667} r'(t)\,dt$

85. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

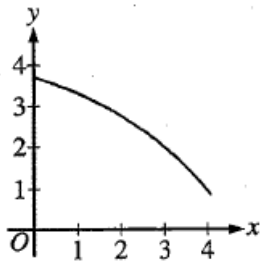
(A)



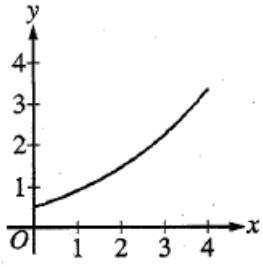
(B)



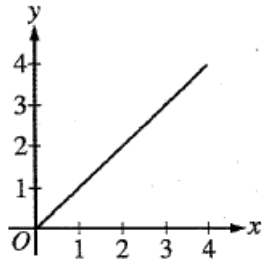
(C)



(D)

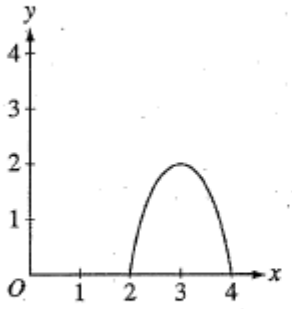


(E)

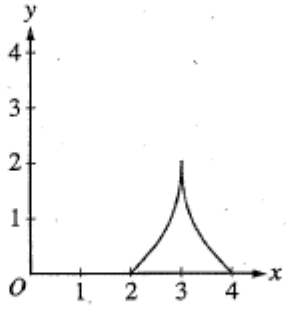


88. On the closed interval $[2, 4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

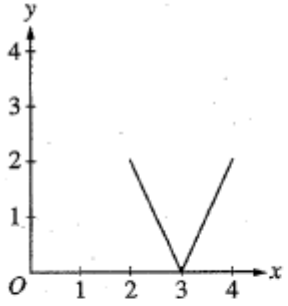
(A)



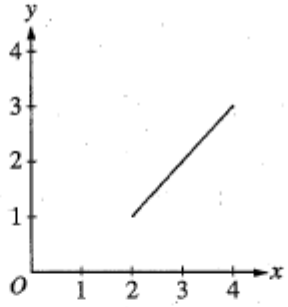
(B)



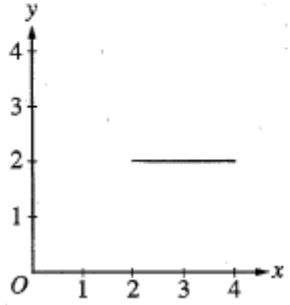
(C)



(D)



(E)



92. Let g be the function given by $g(x) = \int_0^x \sin\left(t^2\right)dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$

(B) $0 \le x \le 1.772$

(C) $1.253 \le x \le 2.171$

(D) $1.772 \le x \le 2.507$

(E) $2.802 \le x \le 3$

2013 Calculator #1

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

2013 NO Calculator #3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

2011 Calculator #2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

2010 Calculator #1

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

   (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

   (b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

   (c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

   (d) How many cubic feet of snow are on the driveway at 9 A.M.?

2010 Calculator #2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.