Mrs. Marcelino

Name: ______________________
Date: ________ Period: _____

Practice: Mixed Unit 1 & 2

Part 1: Basic concepts (Level 2)

1. State each differentiation rule both in symbols and in words.
   (a) The Power Rule
   (c) The Sum Rule
   (d) The Difference Rule
   (e) The Product Rule
   (f) The Quotient Rule
   (g) The Chain Rule

2. State the derivative of each function.
   (a) $y = x^n$ (b) $y = e^x$ (c) $y = a^x$
   (d) $y = \ln x$ (e) $y = \log_a x$ (f) $y = \sin x$
   (g) $y = \cos x$ (h) $y = \tan x$ (i) $y = \csc x$
   (j) $y = \sec x$ (k) $y = \cot x$ (l) $y = \sin^{-1}x$
   (m) $y = \cos^{-1}x$ (n) $y = \tan^{-1}x$

Part 2: True/False (Level 4 if you can justify)

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f and g are differentiable, then
$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

2. If f and g are differentiable, then
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$$

3. If f and g are differentiable, then
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

4. If f is differentiable, then $\dfrac{d}{dx}\sqrt{f(x)} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

5. If f is differentiable, then $\dfrac{d}{dx}f(\sqrt{x}) = \dfrac{f'(x)}{2\sqrt{x}}$.

6. If $y = e^2$, then $y' = 2e$.

7. $\dfrac{d}{dx}(10^x) = x10^{x-1}$

8. $\dfrac{d}{dx}(\ln 10) = \dfrac{1}{10}$

9. $\dfrac{d}{dx}(\tan^2 x) = \dfrac{d}{dx}(\sec^2 x)$

10. $\dfrac{d}{dx}|x^2 + x| = |2x + 1|$

11. If $g(x) = x^5$, then $\lim\limits_{x \to 2} \dfrac{g(x) - g(2)}{x - 2} = 80$.

12. An equation of the tangent line to the parabola $y = x^2$ at $(-2, 4)$ is $y - 4 = 2x(x + 2)$.

Part 3: Problems – Mixed concepts

1–50 Calculate y'.

1. $y = (x^4 - 3x^2 + 5)^3$
2. $y = \cos(\tan x)$
3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$
4. $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$
5. $y = 2x\sqrt{x^2 + 1}$
6. $y = \dfrac{e^x}{1 + x^2}$
7. $y = e^{\sin 2\theta}$
8. $y = e^{-t}(t^2 - 2t + 2)$
9. $y = \dfrac{t}{1 - t^2}$
10. $y = e^{mx}\cos nx$
11. $y = \sqrt{x}\cos\sqrt{x}$
12. $y = (\arcsin 2x)^2$
13. $y = \dfrac{e^{1/x}}{x^2}$
14. $y = \dfrac{1}{\sin(x - \sin x)}$
15. $xy^4 + x^2y = x + 3y$
16. $y = \ln(\csc 5x)$
17. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta}$
18. $x^2\cos y + \sin 2y = xy$
19. $y = e^{cx}(c\sin x - \cos x)$
20. $y = \ln(x^2e^x)$
21. $y = 3^{x\ln x}$
22. $y = \sec(1 + x^2)$
23. $y = (1 - x^{-1})^{-1}$
24. $y = 1/\sqrt[3]{x + \sqrt{x}}$
25. $\sin(xy) = x^2 - y$
26. $y = \sqrt{\sin\sqrt{x}}$
27. $y = \log_5(1 + 2x)$
28. $y = (\cos x)^x$
29. $y = \ln\sin x - \frac{1}{2}\sin^2 x$
30. $y = \dfrac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$
31. $y = x\tan^{-1}(4x)$
32. $y = e^{\cos x} + \cos(e^x)$
33. $y = \ln|\sec 5x + \tan 5x|$
34. $y = 10^{\tan \pi\theta}$
35. $y = \cot(3x^2 + 5)$
36. $y = \sqrt{t\ln(t^4)}$
37. $y = \sin(\tan\sqrt{1 + x^3})$
38. $y = \arctan(\arcsin\sqrt{x})$
39. $y = \tan^2(\sin\theta)$
40. $xe^y = y - 1$
41. $y = \dfrac{\sqrt{x + 1}\,(2 - x)^5}{(x + 3)^7}$
42. $y = \dfrac{(x + \lambda)^4}{x^4 + \lambda^4}$
44. $y = \dfrac{\sin mx}{x}$
46. $y = \ln\left|\dfrac{x^2 - 4}{2x + 5}\right|$
49. $y = \cos(e^{\sqrt{\tan 3x}})$
50. $y = \sin^2(\cos\sqrt{\sin \pi x})$
51. If $f(t) = \sqrt{4t + 1}$, find $f''(2)$.
52. If $g(\theta) = \theta\sin\theta$, find $g''(\pi/6)$.
53. Find y'' if $x^6 + y^6 = 1$.
54. Find $f^{(n)}(x)$ if $f(x) = 1/(2 - x)$.

56. Evaluate $\lim_{t \to 0} \dfrac{t^3}{\tan^3(2t)}$.

57–59 Find an equation of the tangent to the curve at the given point.

57. $y = 4\sin^2 x$, $(\pi/6, 1)$

58. $y = \dfrac{x^2 - 1}{x^2 + 1}$, $(0, -1)$

59. $y = \sqrt{1 + 4\sin x}$, $(0, 1)$

60–61 Find equations of the tangent line and normal line to the curve at the given point.

60. $x^2 + 4xy + y^2 = 13$, $(2, 1)$

61. $y = (2 + x)e^{-x}$, $(0, 2)$

63. (a) If $f(x) = x\sqrt{5 - x}$, find $f'(x)$.
    (b) Find equations of the tangent lines to the curve $y = x\sqrt{5 - x}$ at the points $(1, 2)$ and $(4, 4)$.
    (c) Illustrate part (b) by graphing the curve and tangent lines on the same screen.

65. At what points on the curve $y = \sin x + \cos x$, $0 \leqslant x \leqslant 2\pi$, is the tangent line horizontal?

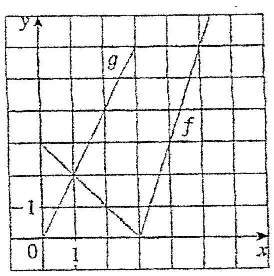
66. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

67. If $f(x) = (x - a)(x - b)(x - c)$, show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x - a} + \frac{1}{x - b} + \frac{1}{x - c}$$

Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$, and $f'(5) = 11$. Find (a) $h'(2)$ and (b) $F'(2)$.

70. If f and g are the functions whose graphs are shown, let $P(x) = f(x)g(x)$, $Q(x) = f(x)/g(x)$, and $C(x) = f(g(x))$. Find (a) $P'(2)$, (b) $Q'(2)$, and (c) $C'(2)$.



71–78 Find f' in terms of g'.

71. $f(x) = x^2 g(x)$

72. $f(x) = g(x^2)$

73. $f(x) = [g(x)]^2$

74. $f(x) = g(g(x))$

75. $f(x) = g(e^x)$

76. $f(x) = e^{g(x)}$

77. $f(x) = \ln\big(g(x)\big)$

78. $f(x) = g(\ln x)$

79–81 Find h' in terms of f' and g'.

79. $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$

80. $h(x) = \sqrt{\dfrac{f(x)}{g(x)}}$

81. $h(x) = f(g(\sin 4x))$

83. At what point on the curve $y = [\ln(x + 4)]^2$ is the tangent horizontal?

84. (a) Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$.
    (b) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

85. Find a parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2, respectively.

86. The function $C(t) = K(e^{-at} - e^{-bt})$, where a, b, and K are positive constants and $b > a$, is used to model the concentration at time t of a drug injected into the bloodstream.
    (a) Show that $\lim_{t \to \infty} C(t) = 0$.
    (b) Find $C'(t)$, the rate at which the drug is cleared from circulation.
    (c) When is this rate equal to 0?

87. An equation of motion of the form $s = Ae^{-ct}\cos(\omega t + \delta)$ represents damped oscillation of an object. Find the velocity and acceleration of the object.

106–108 Express the limit as a derivative and evaluate.

106. $\lim_{x \to 1} \dfrac{x^{17} - 1}{x - 1}$

107. $\lim_{h \to 0} \dfrac{\sqrt[4]{16 + h} - 2}{h}$

108. $\lim_{\theta \to \pi/3} \dfrac{\cos\theta - 0.5}{\theta - \pi/3}$

a) The Power Rule: If n is any real number, then $\frac{d}{dx}(x^n) = nx^{n-1}$. The derivative of a variable base raised to a constant power is the power times the base raised to the power minus one.

b) The Constant Multiple Rule: If c is a constant and f is a differentiable function, then $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$.

The derivative of a constant times a function is the constant times the derivative of the function.

c) The Sum Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$. The derivative of a sum

ions is the sum of the derivatives.

d) The Difference Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$. The derivative of a difference of functions is the difference of the derivatives.

e) The Product Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x)\,g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$. The derivative of a product of two functions is the first function times the derivative of the second function plus the second function times the derivative of the first function.

f) The Quotient Rule: If f and g are both differentiable, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$.

The derivative of a quotient of functions is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

g) The Chain Rule: If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product $F'(x) = f'(g(x))\,g'(x)$. The derivative of a composite function is the derivative of the outer function evaluated at the inner function times the derivative of the inner function.

2. (a) $y = x^n \Rightarrow y' = nx^{n-1}$ (b) $y = e^x \Rightarrow y' = e^x$
(c) $y = a^x \Rightarrow y' = a^x \ln a$ (d) $y = \ln x \Rightarrow y' = 1/x$
(e) $y = \log_a x \Rightarrow y' = 1/(x \ln a)$ (f) $y = \sin x \Rightarrow y' = \cos x$
(g) $y = \cos x \Rightarrow y' = -\sin x$ (h) $y = \tan x \Rightarrow y' = \sec^2 x$
(i) $y = \csc x \Rightarrow y' = -\csc x \cot x$ (j) $y = \sec x \Rightarrow y' = \sec x \tan x$
(k) $y = \cot x \Rightarrow y' = -\csc^2 x$ (l) $y = \sin^{-1} x \Rightarrow y' = 1/\sqrt{1-x^2}$
(m) $y = \cos^{-1} x \Rightarrow y' = -1/\sqrt{1-x^2}$ (n) $y = \tan^{-1} x \Rightarrow y' = 1/(1+x^2)$

1. True. This is the Sum Rule.

2. False. See the warning before the Product Rule.

3. True. This is the Chain Rule.

4. True by the Chain Rule.

5. $\frac{d}{dx}f\left(\sqrt{x}\right) = \frac{f'\left(\sqrt{x}\right)}{2\sqrt{x}}$ by the Chain Rule.

6. False. e^2 is a constant, so $y' = 0$.

7. False. $\frac{d}{dx}10^x = 10^x \ln 10$

8. False. $\ln 10$ is a constant, so its derivative is 0.

9. True. $\frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x$, and $\frac{d}{dx}(\sec^2 x) = 2\sec x(\sec x \tan x) = 2\tan x \sec^2 x$.
Or: $\frac{d}{dx}(\sec^2 x) = \frac{d}{dx}(1+\tan^2 x) = \frac{d}{dx}(\tan^2 x)$.

10. False. $f(x) = |x^2+x| = x^2+x$ for $x \geq 0$ or $x \leq -1$ and $|x^2+x| = -(x^2+x)$ for $-1 < x < 0$.
So $f'(x) = 2x+1$ for $x > 0$ or $x < -1$ and $f'(x) = -(2x+1)$ for $-1 < x < 0$. But $|2x+1| = 2x+1$ for $x \geq -\frac{1}{2}$ and $|2x+1| = -2x-1$ for $x < -\frac{1}{2}$.

11. True. $g(x) = x^5 \Rightarrow g'(x) = 5x^4 \Rightarrow g'(2) = 5(2)^4 = 80$, and by the definition of the derivative,
$\lim_{x\to 2}\frac{g(x)-g(2)}{x-2} = g'(2) = 80$.

12. False. A tangent line to the parabola $y = x^2$ has slope $dy/dx = 2x$, so at $(-2,4)$ the slope of the tangent is $2(-2) = -4$ and an equation of the tangent line is $y - 4 = -4(x+2)$. [The given equation, $y - 4 = 2x(x+2)$, is not even linear!]

1. $y = (x^4 - 3x^2 + 5)^3 \Rightarrow$
$y' = 3(x^4-3x^2+5)^2\frac{d}{dx}(x^4-3x^2+5) = 3(x^4-3x^2+5)^2(4x^3-6x) = 6x(x^4-3x^2+5)^2(2x^2-3)$

2. $y = \cos(\tan x) \Rightarrow y' = -\sin(\tan x)\frac{d}{dx}(\tan x) = -\sin(\tan x)(\sec^2 x)$

3. $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \Rightarrow y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}}$

4. $y = \frac{3x-2}{\sqrt{2x+1}} \Rightarrow$
$y' = \frac{\sqrt{2x+1}(3) - (3x-2)\frac{1}{2}(2x+1)^{-1/2}(2)}{\left(\sqrt{2x+1}\right)^2} \cdot \frac{(2x+1)^{1/2}}{(2x+1)^{1/2}} = \frac{3(2x+1)-(3x-2)}{(2x+1)^{3/2}} = \frac{3x+5}{(2x+1)^{3/2}}$

$= 2x\sqrt{x^2+1} \Rightarrow$
$y' = 2x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + \sqrt{x^2+1}\,(2) = \frac{2x^2}{\sqrt{x^2+1}} + 2\sqrt{x^2+1} = \frac{2x^2+2(x^2+1)}{\sqrt{x^2+1}} = \frac{2(2x^2+1)}{\sqrt{x^2+1}}$

6. $y = \frac{e^x}{1+x^2} \Rightarrow y' = \frac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \frac{e^x(x^2-2x+1)}{(1+x^2)^2} = \frac{e^x(x-1)^2}{(1+x^2)^2}$

7. $y = e^{\sin 2\theta} \Rightarrow y' = e^{\sin 2\theta}\frac{d}{d\theta}(\sin 2\theta) = e^{\sin 2\theta}(\cos 2\theta)(2) = 2\cos 2\theta\, e^{\sin 2\theta}$

8. $y = e^{-t}(t^2-2t+2) \Rightarrow y' = e^{-t}(2t-2) + (t^2-2t+2)(-e^{-t}) = e^{-t}(2t-2-t^2+2t-2) = e^{-t}(-t^2+4t-4)$

9. $y = \frac{t}{1-t^2} \Rightarrow y' = \frac{(1-t^2)(1) - t(-2t)}{(1-t^2)^2} = \frac{1-t^2+2t^2}{(1-t^2)^2} = \frac{t^2+1}{(1-t^2)^2}$

10. $y = e^{mx}\cos nx \Rightarrow$
$y' = e^{mx}(\cos nx)' + \cos nx\,(e^{mx})' = e^{mx}(-\sin nx \cdot n) + \cos nx\,(e^{mx}\cdot m) = e^{mx}(m\cos nx - n\sin nx)$

11. $y = \sqrt{x}\cos\sqrt{x} \Rightarrow$
$y' = \sqrt{x}\left(\cos\sqrt{x}\right)' + \cos\sqrt{x}\left(\sqrt{x}\right)' = \sqrt{x}\left[-\sin\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)\right] + \cos\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)$
$= \frac{1}{2}x^{-1/2}\left(-\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right) = \frac{\cos\sqrt{x} - \sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}$

12. $y = (\arcsin 2x)^2 \Rightarrow y' = 2(\arcsin 2x)\cdot(\arcsin 2x)' = 2\arcsin 2x \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{4\arcsin 2x}{\sqrt{1-4x^2}}$

13. $y = \frac{e^{1/x}}{x^2} \Rightarrow y' = \frac{x^2(e^{1/x})' - e^{1/x}(x^2)'}{(x^2)^2} = \frac{x^2(e^{1/x})(-1/x^2) - e^{1/x}(2x)}{x^4} = \frac{-e^{1/x}(1+2x)}{x^4}$

14. Using the Reciprocal Rule, $g(x) = \frac{1}{f(x)} \Rightarrow g'(x) = -\frac{f'(x)}{[f(x)]^2}$, we have $y = \frac{1}{\sin(x-\sin x)} \Rightarrow$
$y' = -\frac{\cos(x-\sin x)(1-\cos x)}{\sin^2(x-\sin x)}$.

15. $\frac{d}{dx}(xy^4 + x^2y) = \frac{d}{dx}(x+3y) \Rightarrow x\cdot 4y^3y' + y^4\cdot 1 + x^2\cdot y' + y\cdot 2x = 1 + 3y' \Rightarrow$
$y'(4xy^3+x^2-3) = 1 - y^4 - 2xy \Rightarrow y' = \frac{1-y^4-2xy}{4xy^3+x^2-3}$

16. $y = \ln(\csc 5x) \Rightarrow y' = \frac{1}{\csc 5x}(-\csc 5x\cot 5x)(5) = -5\cot 5x$

17. $y = \frac{\sec 2\theta}{1+\tan 2\theta} \Rightarrow$
$y' = \frac{(1+\tan 2\theta)(\sec 2\theta\tan 2\theta\cdot 2) - (\sec 2\theta)(\sec^2 2\theta\cdot 2)}{(1+\tan 2\theta)^2} = \frac{2\sec 2\theta\,[(1+\tan 2\theta)\tan 2\theta - \sec^2 2\theta]}{(1+\tan 2\theta)^2}$
$= \frac{2\sec 2\theta\,(\tan 2\theta + \tan^2 2\theta - \sec^2 2\theta)}{(1+\tan 2\theta)^2} = \frac{2\sec 2\theta\,(\tan 2\theta - 1)}{(1+\tan 2\theta)^2}$ $[1 + \tan^2 x = \sec^2 x]$

18. $\frac{d}{dx}(x^2\cos y + \sin 2y) = \frac{d}{dx}(xy) \Rightarrow x^2(-\sin y\cdot y') + (\cos y)(2x) + \cos 2y\cdot 2y' = x\cdot y' + y\cdot 1 \Rightarrow$
$y'(-x^2\sin y + 2\cos 2y - x) = y - 2x\cos y \Rightarrow y' = \frac{y-2x\cos y}{2\cos 2y - x^2\sin y - x}$

19. $y = e^{cx}(c\sin x - \cos x) \Rightarrow$
$y' = e^{cx}(c\cos x + \sin x) + ce^{cx}(c\sin x - \cos x) = e^{cx}(c^2\sin x - c\cos x + c\cos x + \sin x)$
$= e^{cx}(c^2\sin x + \sin x) = e^{cx}\sin x\,(c^2+1)$

20. $y = \ln(x^2e^x) = \ln x^2 + \ln e^x = 2\ln x + x \Rightarrow y' = 2/x + 1$

21. $y = 3^{x\ln x} \Rightarrow y' = 3^{x\ln x}(\ln 3)\frac{d}{dx}(x\ln x) = 3^{x\ln x}(\ln 3)\left(x\cdot\frac{1}{x} + \ln x\cdot 1\right) = 3^{x\ln x}(\ln 3)(1+\ln x)$

22. $y = \sec(1+x^2) \Rightarrow y' = 2x\sec(1+x^2)\tan(1+x^2)$

23. $y = (1-x^{-1})^{-1} \Rightarrow$
$y' = -1(1-x^{-1})^{-2}[-(-1x^{-2})] = -(1-1/x)^{-2}x^{-2} = -((x-1)/x)^{-2}x^{-2} = -(x-1)^{-2}$

24. $y = \left(x+\sqrt{x}\right)^{-1/3} \Rightarrow y' = -\frac{1}{3}\left(x+\sqrt{x}\right)^{-4/3}\left(1+\frac{1}{2\sqrt{x}}\right)$

25. $\sin(xy) = x^2 - y \Rightarrow \cos(xy)(xy' + y\cdot 1) = 2x - y' \Rightarrow x\cos(xy)y' + y' = 2x - y\cos(xy) \Rightarrow$
$y'[x\cos(xy)+1] = 2x - y\cos(xy) \Rightarrow y' = \frac{2x-y\cos(xy)}{x\cos(xy)+1}$

26. $y = \sqrt{\sin\sqrt{x}} \Rightarrow y' = \frac{1}{2}\left(\sin\sqrt{x}\right)^{-1/2}\left(\cos\sqrt{x}\right)\left(\frac{1}{2\sqrt{x}}\right) = \frac{\cos\sqrt{x}}{4\sqrt{x\sin\sqrt{x}}}$

27. $y = \log_5(1+2x) \Rightarrow y' = \frac{1}{(1+2x)\ln 5}\frac{d}{dx}(1+2x) = \frac{2}{(1+2x)\ln 5}$

28. $y = (\cos x)^x \Rightarrow \ln y = \ln(\cos x)^x = x\ln\cos x \Rightarrow \frac{y'}{y} = x\cdot\frac{1}{\cos x}\cdot(-\sin x) + \ln\cos x\cdot 1 \Rightarrow$
$y' = (\cos x)^x(\ln\cos x - x\tan x)$

29. $y = \ln\sin x - \frac{1}{2}\sin^2 x \Rightarrow y' = \frac{1}{\sin x}\cdot\cos x - \frac{1}{2}\cdot 2\sin x\cdot\cos x = \cot x - \sin x\cos x$

30. $y = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} \Rightarrow$
$\ln y = \ln\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} = \ln(x^2+1)^4 - \ln[(2x+1)^3(3x-1)^5] = 4\ln(x^2+1) - [\ln(2x+1)^3 + \ln(3x-1)^5]$
$= 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1) \Rightarrow$
$\frac{y'}{y} = 4\cdot\frac{1}{x^2+1}\cdot 2x - 3\cdot\frac{1}{2x+1}\cdot 2 - 5\cdot\frac{1}{3x-1}\cdot 3 \Rightarrow y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left(\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right)$.
[*The answer could be simplified to* $y' = -\frac{(x^2+56x+9)(x^2+1)^3}{(2x+1)^4(3x-1)^6}$, *but this is unnecessary.*]

31. $y = x\tan^{-1}(4x) \Rightarrow y' = x\cdot\frac{1}{1+(4x)^2}\cdot 4 + \tan^{-1}(4x)\cdot 1 = \frac{4x}{1+16x^2} + \tan^{-1}(4x)$

32. $y = e^{\cos x} + \cos(e^x) \Rightarrow y' = e^{\cos x}(-\sin x) + [-\sin(e^x)\cdot e^x] = -\sin x\, e^{\cos x} - e^x\sin(e^x)$

33. $y = \ln|\sec 5x + \tan 5x| \Rightarrow$
$y' = \frac{1}{\sec 5x + \tan 5x}(\sec 5x\tan 5x\cdot 5 + \sec^2 5x\cdot 5) = \frac{5\sec 5x\,(\tan 5x + \sec 5x)}{\sec 5x + \tan 5x} = 5\sec 5x$

34. $y = 10^{\tan\pi\theta} \Rightarrow y' = 10^{\tan\pi\theta}\cdot\ln 10\cdot\sec^2\pi\theta\cdot\pi = \pi(\ln 10)10^{\tan\pi\theta}\sec^2\pi\theta$

35. $y = \cot(3x^2+5) \Rightarrow y' = -\csc^2(3x^2+5)(6x) = -6x\csc^2(3x^2+5)$

36) $y = \sqrt{t\ln(t^4)} \Rightarrow y = (t\ln(t^4))^{\frac{1}{2}}$

$\Rightarrow y' = \left(\ln(t^4) + t \cdot 4t^3 \cdot \frac{1}{t^4}\right) \cdot \frac{1}{2}\left(t\ln(t^4)\right)^{-\frac{1}{2}}$

$= \dfrac{(\ln(t^4) + 4)}{2(t\ln(t^4))^{\frac{1}{2}}} = \dfrac{\ln(t^4)+4}{2\sqrt{t\ln(t^4)}}$

$= \dfrac{4\ln t + 4}{2\sqrt{4t\ln}} = \dfrac{\ln t + 1}{\sqrt{t \ln t}}$

37) $y = \sin(\tan\sqrt{1+x^3}) \Rightarrow y = \sin(\tan(1+x^3)^{\frac{1}{2}})$

$\Rightarrow y' = 3x^2 \cdot \frac{1}{2}(1+x^3)^{-\frac{1}{2}} \cdot \sec^2(1+x^3)^{\frac{1}{2}} \cdot \cos(\tan(1+x^3)^{\frac{1}{2}})$

$= \dfrac{3x^2 \sec^2\sqrt{1+x^3}\cos(\tan\sqrt{1+x^3})}{2\sqrt{1+x^3}}$

38) $y = \arctan(\arcsin\sqrt{x}) \Rightarrow y = \arctan(\arcsin x^{\frac{1}{2}})$

$y' \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} \cdot \dfrac{1}{\sqrt{1-(x^{\frac{1}{2}})^2}} \cdot \dfrac{1}{(\arcsin x^{\frac{1}{2}})^2+1}$

$\Rightarrow \dfrac{1}{2\sqrt{x}\sqrt{1-x}\left((\arcsin\sqrt{x})^2+1\right)}$

$\Rightarrow \dfrac{1}{2\sqrt{x-x^2}\left((\arcsin\sqrt{x})^2+1\right)}$

39) $y = \tan^2(\sin\theta) \Rightarrow y = (\tan(\sin\theta))^2$

$y' = \cos\theta \cdot \sec^2(\sin\theta) \cdot 2(\tan(\sin\theta))$

40) $xe^y = y-1 \Rightarrow e^y + xy'e^y = y' \Rightarrow e^y = -xy'e^y + y'$

$\Rightarrow e^y = y'(-xe^y+1) \Rightarrow \dfrac{e^y}{-xe^y+1} = y'$

41) $y' = \dfrac{(x+3)^7\left[\sqrt{x+1} \cdot -1 \cdot 5(2-x)^4 + (2-x)^5 \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}}\right] - \sqrt{x+1}(2-x)^5 \cdot 7(x+3)^6}{(x+3)^{14}}$

$y' = \dfrac{(x+3)^7\left(-5\sqrt{x+1}(2-x)^4 + \frac{1}{2}(2-x)^5(x+1)^{-\frac{1}{2}}\right) - 7\sqrt{x+1}(2-x)^5(x+3)^6}{(x+3)^{14}}$

$y' = \dfrac{(x+3)^6(2-x)^4(x+1)^{-\frac{1}{2}}\left[(x+3)\left(-5(x+1)+\frac{1}{2}(2-x)\right) - 7(x+1)(2-x)\right]}{(x+3)^{14}}$

$= \dfrac{(2-x)^4\left[(x+3)(-\frac{11}{2}x-4) - 7(x+1)(2-x)\right]}{(x+3)^8\sqrt{x+1}}$

$= \dfrac{(2-x)^4\left[\frac{1}{2}(3x^2-55x-52)\right]}{(x+3)^8\sqrt{x+1}} = \dfrac{(2-x)^4(3x^2-55x-52)}{2(x+3)^8\sqrt{x+1}}$

42. $y' = \dfrac{(x^4+\lambda^4) \cdot 4(x+\lambda)^3 - (x+\lambda)^4 \cdot 4x^3}{(x^4+\lambda^4)^2}$

$= \dfrac{4(x+\lambda)^3\left[(x^4+\lambda^4) - (x+\lambda)x^3\right]}{(x^4+\lambda^4)^2}$

$= \dfrac{4(x+\lambda)^3\left[\lambda^4 - \lambda x^3\right]}{(x^4+\lambda^4)^2}$

44. $y' = \dfrac{x \cdot m\cos mx - \sin mx}{x^2}$

46. option 1: $\left(\dfrac{(2x+5)\cdot 2x - (x^2-4)\cdot 2}{(2x+5)^2}\right) \cdot \dfrac{1}{\left(\frac{x^2-4}{2x+5}\right)}$

$= \dfrac{2x^2+10x+8}{(2x+5)^2} \cdot \dfrac{(2x+5)}{x^2-4} = \dfrac{2(x+1)(x+4)}{(2x+5)(x^2-4)}$

option 2: Rewrite $y = \ln(x^2-4) - \ln(2x+5)$

$y' = 2x \cdot \dfrac{1}{x^2-4} - 2 \cdot \dfrac{1}{2x+5}$

$= \dfrac{2x}{x^2-4} - \dfrac{2}{2x+5} = \dfrac{2x(2x+5) - 2(x^2-4)}{(x^2-4)(2x+5)}$

$= \dfrac{2(x+1)(x+4)}{(2x+5)(x^2-4)}$

49. $y = \cos(e^{(\tan(3x))^{\frac{1}{2}}})$

$y' = 3 \cdot \sec^2(3x) \cdot \frac{1}{2}(\tan(3x))^{-\frac{1}{2}} \cdot e^{(\tan(3x))^{\frac{1}{2}}} \cdot (-\sin(e^{\tan(3x)^{\frac{1}{2}}}))$

$= \dfrac{-3\sec^2(3x) \cdot e^{\sqrt{\tan 3x}} \cdot \sin(e^{\sqrt{\tan 3x}})}{2\sqrt{\tan(3x)}}$

50. $y = \sin^2\left(\cos\sqrt{\sin \pi x}\right) = \left[\sin\left(\cos\sqrt{\sin\pi x}\right)\right]^2 \Rightarrow$

$y' = 2\left[\sin\left(\cos\sqrt{\sin\pi x}\right)\right]\left[\sin\left(\cos\sqrt{\sin\pi x}\right)\right]' = 2\sin\left(\cos\sqrt{\sin\pi x}\right)\cos\left(\cos\sqrt{\sin\pi x}\right)\left(\cos\sqrt{\sin\pi x}\right)'$

$= 2\sin\left(\cos\sqrt{\sin\pi x}\right)\cos\left(\cos\sqrt{\sin\pi x}\right)\left(-\sin\sqrt{\sin\pi x}\right)\left(\sqrt{\sin\pi x}\right)'$

$= -2\sin\left(\cos\sqrt{\sin\pi x}\right)\cos\left(\cos\sqrt{\sin\pi x}\right)\sin\sqrt{\sin\pi x} \cdot \frac{1}{2}(\sin\pi x)^{-1/2}(\sin\pi x)'$

$= \dfrac{-\sin\left(\cos\sqrt{\sin\pi x}\right)\cos\left(\cos\sqrt{\sin\pi x}\right)\sin\sqrt{\sin\pi x}}{\sqrt{\sin\pi x}} \cdot \cos\pi x \cdot \pi$

$= \dfrac{-\pi\sin\left(\cos\sqrt{\sin\pi x}\right)\cos\left(\cos\sqrt{\sin\pi x}\right)\sin\sqrt{\sin\pi x}\cos\pi x}{\sqrt{\sin\pi x}}$

51. $f(t) = \sqrt{4t+1} \Rightarrow f'(t) = \frac{1}{2}(4t+1)^{-1/2} \cdot 4 = 2(4t+1)^{-1/2} \Rightarrow$

$f''(t) = 2(-\frac{1}{2})(4t+1)^{-3/2} \cdot 4 = -4/(4t+1)^{3/2}$, so $f''(2) = -4/9^{3/2} = -\frac{4}{27}$.

52. $g(\theta) = \theta\sin\theta \Rightarrow g'(\theta) = \theta\cos\theta + \sin\theta \cdot 1 \Rightarrow g''(\theta) = \theta(-\sin\theta) + \cos\theta \cdot 1 + \cos\theta = 2\cos\theta - \theta\sin\theta$,

so $g''(\pi/6) = 2\cos(\pi/6) - (\pi/6)\sin(\pi/6) = 2(\sqrt{3}/2) - (\pi/6)(1/2) = \sqrt{3} - \pi/12$.

53. $x^6 + y^6 = 1 \Rightarrow 6x^5 + 6y^5y' = 0 \Rightarrow y' = -x^5/y^5 \Rightarrow$

$y'' = -\dfrac{y^5(5x^4) - x^5(5y^4y')}{(y^5)^2} = -\dfrac{5x^4y^4\left[y - x(-x^5/y^5)\right]}{y^{10}} = -\dfrac{5x^4\left[(y^6+x^6)/y^5\right]}{y^6} = -\dfrac{5x^4}{y^{11}}$

54. $f(x) = (2-x)^{-1} \Rightarrow f'(x) = (2-x)^{-2} \Rightarrow f''(x) = 2(2-x)^{-3} \Rightarrow f'''(x) = 2 \cdot 3(2-x)^{-4} \Rightarrow$

$f^{(4)}(x) = 2 \cdot 3 \cdot 4(2-x)^{-5}$. In general, $f^{(n)}(x) = 2 \cdot 3 \cdot 4 \cdot \cdots \cdot n(2-x)^{-(n+1)} = \dfrac{n!}{(2-x)^{(n+1)}}$.

56. $\displaystyle\lim_{t\to 0}\frac{t^3}{\tan^3 2t} = \lim_{t\to 0}\frac{t^3\cos^3 2t}{\sin^3 2t} = \lim_{t\to 0}\cos^3 2t \cdot \frac{1}{8\frac{\sin^3 2t}{(2t)^3}} = \lim_{t\to 0}\frac{\cos^3 2t}{8\left(\lim_{t\to 0}\frac{\sin 2t}{2t}\right)^3} = \frac{1}{8 \cdot 1^3} = \frac{1}{8}$

57. $y = 4\sin^2 x \Rightarrow y' = 4 \cdot 2\sin x\cos x$. At $(\frac{\pi}{6}, 1)$, $y' = 8 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$, so an equation of the tangent line is $y - 1 = 2\sqrt{3}(x - \frac{\pi}{6})$, or $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$.

58. $y = \dfrac{x^2-1}{x^2+1} \Rightarrow y' = \dfrac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$.

At $(0,-1)$, $y' = 0$, so an equation of the tangent line is $y + 1 = 0(x-0)$, or $y = -1$.

59. $y = \sqrt{1+4\sin x} \Rightarrow y' = \frac{1}{2}(1+4\sin x)^{-1/2} \cdot 4\cos x = \dfrac{2\cos x}{\sqrt{1+4\sin x}}$.

At $(0,1)$, $y' = \dfrac{2}{\sqrt{1}} = 2$, so an equation of the tangent line is $y - 1 = 2(x-0)$, or $y = 2x+1$.

60. $x^2 + 4xy + y^2 = 13 \Rightarrow 2x + 4(xy' + y \cdot 1) + 2yy' = 0 \Rightarrow x + 2xy' + 2y + yy' = 0 \Rightarrow$

$2xy' + yy' = -x - 2y \Rightarrow y'(2x+y) = -x-2y \Rightarrow y' = \dfrac{-x-2y}{2x+y}$.

At $(2,1)$, $y' = \dfrac{-2-2}{4+1} = -\dfrac{4}{5}$, so an equation of the tangent line is $y - 1 = -\frac{4}{5}(x-2)$, or $y = -\frac{4}{5}x + \frac{13}{5}$.

The slope of the normal line is $\frac{5}{4}$, so an equation of the normal line is $y - 1 = \frac{5}{4}(x-2)$, or $y = \frac{5}{4}x - \frac{3}{2}$.

61. $y = (2+x)e^{-x} \Rightarrow y' = (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x)+1] = e^{-x}(-x-1)$.

At $(0,2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x-0)$, or $y = -x+2$.

The slope of the normal line is 1, so an equation of the normal line is $y - 2 = 1(x-0)$, or $y = x+2$.

63. (a) $f(x) = x\sqrt{5-x} \Rightarrow$

$$f'(x) = x\left[\frac{1}{2}(5-x)^{-1/2}(-1)\right] + \sqrt{5-x} = \frac{-x}{2\sqrt{5-x}} + \sqrt{5-x}\cdot\frac{2\sqrt{5-x}}{2\sqrt{5-x}} = \frac{-x}{2\sqrt{5-x}} + \frac{2(5-x)}{2\sqrt{5-x}}$$

$$= \frac{-x+10-2x}{2\sqrt{5-x}} = \frac{10-3x}{2\sqrt{5-x}}$$

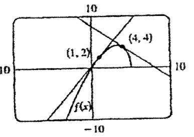
(b) At $(1,2)$: $f'(1) = \frac{7}{4}$.

So an equation of the tangent line is $y - 2 = \frac{7}{4}(x-1)$ or $y = \frac{7}{4}x + \frac{1}{4}$.

$(4,4)$: $f'(4) = -\frac{2}{2} = -1$.

So an equation of the tangent line is $y - 4 = -1(x-4)$ or $y = -x + 8$.

(c)



65. $y = \sin x + \cos x \Rightarrow y' = \cos x - \sin x = 0 \Leftrightarrow \cos x = \sin x$ and $0 \le x \le 2\pi \Leftrightarrow x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$, so the points are $\left(\frac{\pi}{4}, \sqrt{2}\right)$ and $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$.

66. $x^2 + 2y^2 = 1 \Rightarrow 2x + 4yy' = 0 \Rightarrow y' = -x/(2y) = 1 \Leftrightarrow x = -2y$. Since the points lie on the ellipse, we have $(-2y)^2 + 2y^2 = 1 \Rightarrow 6y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{6}}$. The points are $\left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$.

67. $f(x) = (x-a)(x-b)(x-c) \Rightarrow f'(x) = (x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b)$.

So $\dfrac{f'(x)}{f(x)} = \dfrac{(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b)}{(x-a)(x-b)(x-c)} = \dfrac{1}{x-a} + \dfrac{1}{x-b} + \dfrac{1}{x-c}$.

Or: $f(x) = (x-a)(x-b)(x-c) \Rightarrow \ln|f(x)| = \ln|x-a| + \ln|x-b| + \ln|x-c| \Rightarrow$

$$\frac{f'(x)}{f(x)} = \frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c}$$

69. (a) $h(x) = f(x)\,g(x) \Rightarrow h'(x) = f(x)\,g'(x) + g(x)\,f'(x) \Rightarrow$

$h'(2) = f(2)\,g'(2) + g(2)\,f'(2) = (3)(4) + (5)(-2) = 12 - 10 = 2$

(b) $F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x))\,g'(x) \Rightarrow F'(2) = f'(g(2))\,g'(2) = f'(5)(4) = 11\cdot 4 = 44$

70. (a) $P(x) = f(x)\,g(x) \Rightarrow P'(x) = f(x)\,g'(x) + g(x)\,f'(x) \Rightarrow$

$P'(2) = f(2)\,g'(2) + g(2)\,f'(2) = (1)\left(\frac{6-0}{3-0}\right) + (4)\left(\frac{0-3}{3-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2$

(b) $Q(x) = \dfrac{f(x)}{g(x)} \Rightarrow Q'(x) = \dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2} \Rightarrow$

$$Q'(2) = \frac{g(2)\,f'(2) - f(2)\,g'(2)}{[g(2)]^2} = \frac{(4)(-1) - (1)(2)}{4^2} = \frac{-6}{16} = -\frac{3}{8}$$

(c) $C(x) = f(g(x)) \Rightarrow C'(x) = f'(g(x))g'(x) \Rightarrow$

$C'(2) = f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6$

71. $f(x) = x^2 g(x) \Rightarrow f'(x) = x^2 g'(x) + g(x)(2x) = x[xg'(x) + 2g(x)]$

72. $f(x) = g(x^2) \Rightarrow f'(x) = g'(x^2)(2x) = 2xg'(x^2)$

73. $f(x) = [g(x)]^2 \Rightarrow f'(x) = 2[g(x)]\cdot g'(x) = 2g(x)\,g'(x)$

74. $f(x) = g(g(x)) \Rightarrow f'(x) = g'(g(x))\,g'(x)$

75. $f(x) = g(e^x) \Rightarrow f'(x) = g'(e^x)\,e^x$

76. $f(x) = e^{g(x)} \Rightarrow f'(x) = e^{g(x)}g'(x)$

77. $f(x) = \ln|g(x)| \Rightarrow f'(x) = \dfrac{1}{g(x)}g'(x) = \dfrac{g'(x)}{g(x)}$

78. $f(x) = g(\ln x) \Rightarrow f'(x) = g'(\ln x)\cdot\dfrac{1}{x} = \dfrac{g'(\ln x)}{x}$

79. $h(x) = \dfrac{f(x)\,g(x)}{f(x) + g(x)} \Rightarrow$

$$h'(x) = \frac{[f(x) + g(x)]\,[f(x)\,g'(x) + g(x)\,f'(x)] - f(x)\,g(x)\,[f'(x) + g'(x)]}{[f(x) + g(x)]^2}$$

$$= \frac{[f(x)]^2\,g'(x) + f(x)\,g(x)\,f'(x) + f(x)\,g(x)\,g'(x) + [g(x)]^2\,f'(x) - f(x)\,g(x)\,f'(x) - f(x)\,g(x)\,g'(x)}{[f(x) + g(x)]^2}$$

$$= \frac{f'(x)\,[g(x)]^2 + g'(x)\,[f(x)]^2}{[f(x) + g(x)]^2}$$

80. $h(x) = \sqrt{\dfrac{f(x)}{g(x)}} \Rightarrow h'(x) = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{2\sqrt{f(x)/g(x)}\,[g(x)]^2} = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{2[g(x)]^{3/2}\sqrt{f(x)}}$

81. Using the Chain Rule repeatedly, $h(x) = f(g(\sin 4x)) \Rightarrow$

$h'(x) = f'(g(\sin 4x))\cdot\dfrac{d}{dx}(g(\sin 4x)) = f'(g(\sin 4x))\cdot g'(\sin 4x)\cdot\dfrac{d}{dx}(\sin 4x) = f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4)$.

83. $y = [\ln(x+4)]^2 \Rightarrow y' = 2[\ln(x+4)]^1\cdot\dfrac{1}{x+4}\cdot 1 = 2\dfrac{\ln(x+4)}{x+4}$ and $y' = 0 \Leftrightarrow \ln(x+4) = 0 \Leftrightarrow$ $x + 4 = e^0 \Rightarrow x + 4 = 1 \Leftrightarrow x = -3$, so the tangent is horizontal at the point $(-3, 0)$.

84. (a) The line $x - 4y = 1$ has slope $\frac{1}{4}$. A tangent to $y = e^x$ has slope $\frac{1}{4}$ when $y' = e^x = \frac{1}{4} \Rightarrow x = \ln\frac{1}{4} = -\ln 4$. Since $y = e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4, \frac{1}{4}\right)$. Thus, an equation of the tangent line is $y - \frac{1}{4} = \frac{1}{4}(x + \ln 4)$ or $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$.

(b) The slope of the tangent at the point (a, e^a) is $\left.\dfrac{d}{dx}e^x\right|_{x=a} = e^a$. Thus, an equation of the tangent line is $y - e^a = e^a(x - a)$. We substitute $x = 0$, $y = 0$ into this equation, since we want the line to pass through the origin: $0 - e^a = e^a(0 - a) \Leftrightarrow -e^a = e^a(-a) \Leftrightarrow a = 1$. So an equation of the tangent line at the point $(a, e^a) = (1, e)$ is $y - e = e(x - 1)$ or $y = ex$.

85. $y = f(x) = ax^2 + bx + c \Rightarrow f'(x) = 2ax + b$. We know that $f'(-1) = 6$ and $f'(5) = -2$, so $-2a + b = 6$ and $10a + b = -2$. Subtracting the first equation from the second gives $12a = -8 \Rightarrow a = -\frac{2}{3}$. Substituting $-\frac{2}{3}$ for a in the first equation gives $b = \frac{14}{3}$. Now $f(1) = 4 \Rightarrow 4 = a + b + c$, so $c = 4 + \frac{2}{3} - \frac{14}{3} = 0$ and hence, $f(x) = -\frac{2}{3}x^2 + \frac{14}{3}x$.

86. (a) $\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty}[K(e^{-at} - e^{-bt})] = K\lim\limits_{t\to\infty}(e^{-at} - e^{-bt}) = K(0 - 0) = 0$ because $-at \to -\infty$ and $-bt \to -\infty$ as $t \to \infty$.

(b) $C(t) = K(e^{-at} - e^{-bt}) \Rightarrow C'(t) = K(e^{-at}(-a) - e^{-bt}(-b)) = K(-ae^{-at} + be^{-bt})$

(c) $C'(t) = 0 \Leftrightarrow be^{-bt} = ae^{-at} \Leftrightarrow \dfrac{b}{a} = e^{(-a+b)t} \Leftrightarrow \ln\dfrac{b}{a} = (b-a)t \Leftrightarrow t = \dfrac{\ln(b/a)}{b-a}$

87. $s(t) = Ae^{-ct}\cos(\omega t + \delta) \Rightarrow$

$v(t) = s'(t) = A\{e^{-ct}[-\omega\sin(\omega t + \delta)] + \cos(\omega t + \delta)(-ce^{-ct})\} = -Ae^{-ct}[\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)] \Rightarrow$

$a(t) = v'(t) = -A\{e^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta)] + [\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)](-ce^{-ct})\}$

$= -Ae^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c^2\cos(\omega t + \delta)]$

$= -Ae^{-ct}[(\omega^2 - c^2)\cos(\omega t + \delta) - 2c\omega\sin(\omega t + \delta)] = Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega\sin(\omega t + \delta)]$

106. $\lim\limits_{x\to 1}\dfrac{x^{17} - 1}{x - 1} = \left[\dfrac{d}{dx}x^{17}\right]_{x=1} = 17(1)^{16} = 17$

107. $\lim\limits_{h\to 0}\dfrac{\sqrt[4]{16+h} - 2}{h} = \left[\dfrac{d}{dx}\sqrt[4]{x}\right]_{x=16} = \left.\dfrac{1}{4}x^{-3/4}\right|_{x=16} = \dfrac{1}{4\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{32}$

108. $\lim\limits_{\theta\to\pi/3}\dfrac{\cos\theta - 0.5}{\theta - \pi/3} = \left[\dfrac{d}{d\theta}\cos\theta\right]_{\theta=\pi/3} = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

AP Calculus Multiple Choice Problems $\boxed{\text{Unit 2 Derivatives}}$

Directions: Any problem with decimal answer choices may be solved with a calculator.

Part 1:

1. The $\lim\limits_{x\to-\infty} \dfrac{2x-1}{1+2x}$ is

(A) -1 (B) 0 (C) 1
(D) 2 (E) nonexistent

4. Given the equation $A = \dfrac{\sqrt{3}}{4}(5s-1)^2$, what is the instantaneous rate of change of A with respect to s at $s = 1$?

(A) $2\sqrt{3}+5$ (B) $2\sqrt{3}$ (C) $\dfrac{5}{2}\sqrt{3}$
(D) $4\sqrt{3}$ (E) $10\sqrt{3}$

5. What is the $\lim\limits_{x\to\ln 2} g(x)$, if

$$g(x) = \begin{cases} e^x \text{ if } x > \ln 2 \\ 4 - e^x \text{ if } x \le \ln 2 \end{cases}?$$

(A) -2 (B) $\ln 2$ (C) e^2
(D) 2 (E) nonexistent

8. What is $\lim\limits_{\Delta x\to 0} \dfrac{\sin\left(\frac{\pi}{3}+\Delta x\right) - \sin\left(\frac{\pi}{3}\right)}{\Delta x}$?

(A) $-\dfrac{1}{2}$ (B) 0 (C) $\dfrac{1}{2}$
(D) $\dfrac{\sqrt{3}}{2}$ (E) nonexistent

10. If $g(x) = 3\tan^2(2x)$, then $g'\left(\dfrac{\pi}{8}\right)$ is

(A) 6 (B) $6\sqrt{2}$ (C) 12
(D) $12\sqrt{2}$ (E) 24

23. Given the equation $y = 3\sin^2\left(\dfrac{x}{2}\right)$, what is an equation of the tangent line to the graph at $x = \pi$?

(A) $y = 3$
(B) $y = \pi$
(C) $y = \pi + 3$
(D) $y = x - \pi + 3$
(E) $y = 3(x - \pi) + 3$

77. The equation of the normal line to the graph $y = e^{2x}$ at the point where $\dfrac{dy}{dx} = 2$ is

(A) $y = -\dfrac{1}{2}x - 1$
(B) $y = -\dfrac{1}{2}x + 1$
(C) $y = 2x + 1$
(D) $y = -\dfrac{1}{2}\left(x - \dfrac{\ln 2}{2}\right) + 2$
(E) $y = 2\left(x - \dfrac{\ln 2}{2}\right) + 2$

89. At what value(s) of x do the graphs of $y = x^2$ and $y = -\sqrt{x}$ have perpendicular tangent lines?

(A) -1 (B) 0 (C) $\dfrac{1}{4}$
(D) 1 (E) none

90. What is the approximate slope of the tangent to the curve $x^3 + y^3 = xy$ at $x = 1$?

(A) -2.420 (B) -1.325 (C) -1.014
(D) -0.698 (E) 0.267

Prior knowledge (Unit 1)

1, 5

Level 3

8, 23, 90

Level 4

4, 77, 89

Solutions to Part 1:

1)C
4)E
5)D
8)C
10)E
23)A
77)B
89)D
90)C

Part 2:

2. The $\lim\limits_{x\to\infty} \dfrac{x^2+4x-5}{x^3-1}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 5

(D) $-\infty$ (E) ∞

3. What is the $\lim\limits_{x\to -2} f(x)$, if

$$f(x) = \begin{cases} |x-1| & \text{if } x > -2 \\ 2x+7 & \text{if } x \le -2 \end{cases}?$$

(A) -3 (B) 1 (C) 3

(D) 11 (E) nonexistent

6. Given the equation $y = 3e^{-2x}$, what is an equation of the normal line to the graph at $x = \ln 2$?

(A) $y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}$

(B) $y = \frac{2}{3}(x + \ln 2) - \frac{3}{4}$

(C) $y = -\frac{3}{2}(x - \ln 2) + \frac{3}{4}$

(D) $y = -\frac{3}{2}(x - \ln 2) - \frac{3}{4}$

(E) $y = 24(x - \ln 2) + 12$

7. What is the $\lim\limits_{h\to 0} \dfrac{\csc(\pi/4 + h) - \csc(\pi/4)}{h}$?

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 0

(D) $-\frac{\sqrt{2}}{2}$ (E) undefined

14. If $f(x) = 5\cos^2(\pi - x)$, then $f'\left(\frac{\pi}{2}\right)$ is

(A) 0 (B) -5 (C) 5

(D) -10 (E) 10

20. What is the slope of the tangent to the curve $x^3 - y^2 = 1$ at $x = 1$?

(A) $-\frac{3}{2}$ (B) 0 (C) $\frac{3}{2\sqrt{2}}$

(D) $\frac{3}{2}$ (E) undefined

Prior knowledge (unit 1)

2, 3

Level 3

7, 14

Level 4

6, 20

Solutions to Part 2

2) C
3) C
6) A
7) C
14) C
20) B

Part 3:

1. Find the derivative of $y = (x^3 + x)^5$.
   (A) $3x^2 + 1$
   (B) $5(x^3 + x)^4$
   (C) $(15x^2 + 5)(x^3 + x)^4$
   (D) $5(3x^2 + 1)$
   (E) $\frac{1}{6}(x^3 + x)^6$

2. Find the derivative of $y = 6\ln\left(\frac{1}{x^2}\right)$.
   (A) -6
   (B) $-\frac{12}{x^3}$
   (C) $\frac{6}{x}$
   (D) $-\frac{12}{x}$
   (E) $-\frac{12}{x^2}$

3. The derivative of $f(x) = \ln(x^2 + 2x + 1)$ is
   (A) $\frac{2x}{x^2 + 2x + 1}$
   (B) $\frac{2}{x + 1}$
   (C) $\frac{1}{x^2 + 2x + 1}$
   (D) $\frac{1}{x + 1}$
   (E) $\frac{2x + 3}{x^2 + 2x + 1}$

4. Find the derivative of $y = \sin(x^2)$.
   (A) $\cos(x^2)$
   (B) $2\sin x\cos x$
   (C) $2x\sin(x^2)$
   (D) $2x\sin x$
   (E) $2x\cos(x^2)$

5. If $y = \ln(\tan x)$, then $y' =$
   (A) $\frac{2}{\sin 2x}$
   (B) $\sec^2 x$
   (C) $\frac{1}{x\tan x}$
   (D) $\cot x$
   (E) $\sec^2 x\tan x$

6. If $y = e^{5x+5}$, then $y'(0) =$
   (A) e^5
   (B) 1
   (C) $5e^5$
   (D) 5
   (E) $\frac{1}{5}e^5$

7. Find the derivative of $y = e^x\sin x$.
   (A) $e^x\cos x$
   (B) $e^x + \cos x$
   (C) $e^x(\sin x + \cos x)$
   (D) $\ln(\sin x)$
   (E) $e\cos x$

8. If $y = \sqrt{x^2 - 2x}$, then $y' =$
   (A) $\frac{1}{2}(x^2 - 2x)$
   (B) $\frac{1}{2}(x^2 - 2x)^{-\frac{1}{2}}$
   (C) $(x^2 - 2x)^{-\frac{1}{2}}(x - 1)$
   (D) $x - 1$
   (E) $(x^2 - 2x)(x - 1)$

9. If $f(x) = \cos x$ and $g(x) = \sqrt{x}$, the derivative of $f \circ g(x)$ is equal to
   (A) $\cos\left(\frac{1}{2\sqrt{x}}\right)$
   (B) $-\sin\sqrt{x}$
   (C) $\sin\sqrt{x}$
   (D) $-\frac{\sin x}{2\sqrt{x}}$
   (E) $-\frac{\sin\sqrt{x}}{2\sqrt{x}}$

10. The derivative of $y = \tan(x^2)$ is
   (A) $\sec^2(x^2)$
   (B) $2x\sec^2(x^2)$
   (C) $2x\sec(x^2)$
   (D) $\sec(x^2)$
   (E) $2x\sec^2(x^2)\tan(x^2)$

11. If $y = \ln e^{\tan^2 x}$, find $y'\left(\frac{\pi}{4}\right)$.
   (A) -2
   (B) 1
   (C) 2
   (D) $2\sqrt{2}$
   (E) 4

13. $(\arctan 3x)' =$
   (A) $\frac{3}{1 + 3x^2}$
   (B) $\frac{3}{1 + x^2}$
   (C) $\frac{3}{1 + 9x^2}$
   (D) $\frac{1}{1 + 9x^2}$
   (E) $\frac{3x}{1 + 3x^2}$

14. $\frac{d}{dx}(\arcsin(x^2)) =$
   (A) $\frac{x^2}{\sqrt{1 - x^4}}$
   (B) $\frac{2x}{\sqrt{1 - x^4}}$
   (C) $\frac{2x}{\sqrt{1 - x^2}}$
   (D) $\frac{1}{\sqrt{1 - x^4}}$
   (E) $\frac{4x}{\sqrt{1 - x^2}}$

Solutions Part 3

1. (C)
2. (D)
3. (B)
4. (E)
5. (A)
6. (C)
7. (C)
8. (C)
9. (E)
10. (B)
11. (E)
12. (D)
13. (C)
14. (B)

Level 2
1, 3, 4, 5, 8, 13

Level 3
2, 6, 7, 10, 14

Level 4 : 9, 11

Part 4:

1. The derivative of $2x - x^{-2}$ is
(A) $2 + 2x$
(B) $2 - x^{-1}$
(C) $x^2 + 2x^{-3}$
(D) $2 + 2x^{-3}$
(E) $2 + 2x^{-1}$

4. The slope of the line tangent to $f(x) = \tan(x^2)$ at $x = 1$ is
(A) 1.557
(B) 1.851
(C) 3.115
(D) 3.426
(E) 6.851

5. Find the slope of the line tangent to $f(x) = \dfrac{\sin(x)}{x}$ at $x = \dfrac{\pi}{2}$.
(A) $\dfrac{2}{\pi}$
(B) 1
(C) -1
(D) $\dfrac{4}{\pi^2}$
(E) $-\dfrac{4}{\pi^2}$

6. Evaluate the derivative of $f(x) = x + e^{2x}$ at $x = 0$.
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Level 2
1,4,6,7,9

Level 3
5,8,10

7. Find $\dfrac{dy}{dx}$ if $y = 3x(x-2)^3$.
(A) $9(x-2)^2$
(B) $9x(x-2)^2$
(C) $(12x-6)(x-2)^2$
(D) $3(x-2)^3$
(E) $(2x-1)(x-2)^2$

8. Find y' if $y = -\dfrac{\cos(x)}{x}$.
(A) $\dfrac{x\sin(x) - \cos(x)}{x^2}$
(B) $\dfrac{x\sin(x) + \cos(x)}{x^2}$
(C) $\dfrac{\sin(x)}{x}$
(D) $\dfrac{\sin(x) - x}{x^2}$
(E) $\dfrac{x\sin(x) - 1}{x^2}$

9. Find the derivative of $\cos^2 x$.
(A) $\sin^2 x$
(B) $1 - \sin^2 x$
(C) $2\cos(x)$
(D) $-2\sin(x)\cos(x)$
(E) $-2\sin(2x)$

10. Find the value of the derivative of $e^x \sin(x)$ at $x = \pi$.
(A) 0
(B) 1
(C) e
(D) e^π
(E) $-e^\pi$

Part 4 Solutions:

1. (D)

2. (B) $f'(x) = \frac{1}{x} = \frac{1}{3}$; $f''(x) = -\frac{1}{x^2} = -\frac{1}{9}$

3. (D) Let $f(x) = e^x$. Then $\lim\limits_{h\to 0} \dfrac{e^{1+h} - e}{h}$
$$= \lim \frac{f(1+h) - f(1)}{h} = f'(1) = e^1 = e.$$

4. (E) $f'(x) = (\tan(x^2))' = 2x\sec^2(x^2)$ at $x = 1$ is $2\sec^2(1) \approx 2(3.426) = 6.851$.

5. (E) $f'(x) = \dfrac{-x\cos(x) - \sin(x)}{x^2}$ at $x = \frac{\pi}{2}$ is $\dfrac{0-1}{\left(\frac{\pi}{2}\right)^2} = -\dfrac{4}{\pi^2}$.

6. (D) $f'(x) = 1 + 2e^{2x}$ at $x = 0$ is $1 + 2 = 3$.

7. (C) $\dfrac{dy}{dx} = 3(x-2)^3 + 3x(3(x-2)^2)$
$= (3x - 6 + 9x)(x-2)^2$
$= (12x - 6)(x-2)^2$

8. (B) $\dfrac{x\sin(x) + \cos(x)}{x^2}$.
$$y' = \frac{-(-x\sin(x) - \cos(x))}{x^2} = \frac{x\sin(x) + \cos(x)}{x^2}$$

9. (D) $-2\sin(x)\cos(x)$

10. (E) $(e^x\sin(x))' = e^x\sin(x) + e^x\cos(x)$ at $x = \pi$ is $e^\pi(0) + e^\pi(-1) = -e^\pi$.

Part 5:

1. Find the equation of the line tangent to $f(x) = 2x + 2e^x$ at $x = 0$.
(A) $y = 4x + 2$
(B) $y = 2x + 2$
(C) $y = 4x$
(D) $y = 4x - 2$
(E) $y = -\frac{1}{4}x + 2$

Level 3
1, 2, 3

2. Find the equation of the line perpendicular to the line tangent to $f(x) = \ln(3 - 2x)$ at $x = 1$.
(A) $y = -2x + 1$
(B) $y = \frac{1}{2}x + 1$
(C) $y = \frac{1}{2}(x - 1)$
(D) $y = \frac{1}{2}(x + 1)$
(E) $y = -2x + 2$

Level 4
4

3. Find the slope of the line tangent to $f(x) = \frac{1}{x^2 + 1}$ at $x = -1$.
(A) -2
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) undefined

4. The y-intercept of the line tangent to $y = x \sin x$ at $x = \pi$ is
(A) $-\pi$
(B) π
(C) $-\pi^2$
(D) π^2
(E) 1

Part 5 Solutions:

1. (A)
2. (C)
3. (B)
4. (D)

Part 6:

1. If f(x)=5, then f'(x)=
   a. 0
   b. 1
   c. 5
   d. 5x
   e. Undefined
2. If f(x)=5x, then f'(x)=
   a. $-\frac{1}{5}$
   b. 0
   c. $\frac{1}{5}$
   d. 1
   e. 5
3. If f(x)=5x+5, then f'(x)=
   a. -1
   b. 0
   c. 1
   d. 5
   e. Undefined
4. If f(x)=5x-5, f'(x)=
   a. -5
   b. 0
   c. 1
   d. 5
   e. 10

Level 2

1, 2, 3, 4

Answers to Part 6:

1. A
2. E
3. D
4. D