**PRACTICE AP CALC BC TEST 1**

**SECTION 1A (NO CALCULATOR!)**

Name _____ PER_____ DATE_____

**50 Minutes—No Calculator**

*Note*: Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

| Q# | Answer |
|---|---|
| 2 | |
| 4 | |
| 5 | |
| 7 | |
| 8 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 17 | |

| Q# | Answer |
|---|---|
| 20 | |
| 21 | |
| 22 | |
| 79 | |
| 3 | |
| 4 | |
| 5 | |
| 7 | |
| 8 | |
| 9 | |
| 12 | |
| 16 | |
| 6 | |

*Show your work and box your answers in the spaces provided below.*

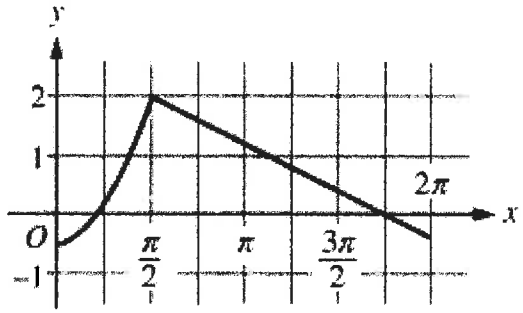
5. Let $f$ be the function defined by $f(x) = e^x \cos x$.

   (a) Find the average rate of change of $f$ on the interval $0 \le x \le \pi$.

   (b) What is the slope of the line tangent to the graph of $f$ at $x = \dfrac{3\pi}{2}$ ?

   (c) Find the absolute minimum value of $f$ on the interval $0 \le x \le 2\pi$. Justify your answer.

   (d) Let $g$ be a differentiable function such that $g\left(\dfrac{\pi}{2}\right) = 0$. The graph of $g'$, the derivative of $g$, is shown below. Find the value of $\lim\limits_{x \to \pi/2} \dfrac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



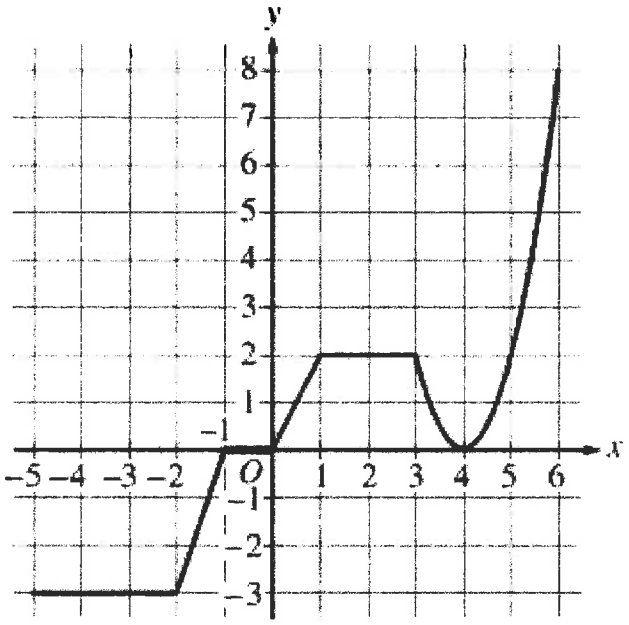

Graph of $g'$

| (a) | (b) |
| --- | --- |
| | |

| (c) | (d) |
| --- | --- |
| | |

*Show your work and box your answers in the spaces provided below.*

**NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.**




Graph of $g$

3. The graph of the continuous function $g$, the derivative of the function $f$, is shown above. The function $g$ is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

   (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of $f$ both increasing and concave up? Give a reason for your answer.

   (d) Find the $x$-coordinate of each point of inflection of the graph of $f$. Give a reason for your answer.

| (c) | (d) |
| --- | --- |
| | |

**50 Minutes—No Calculator**

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) $-1$ (D) $-3$ (E) $-5$

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\dfrac{2}{3}$

(D) $x < \dfrac{2}{3}$ or $x > 2$

(E) $\dfrac{2}{3} < x < 2$

7. $\frac{d}{dx}\cos^2(x^3) =$

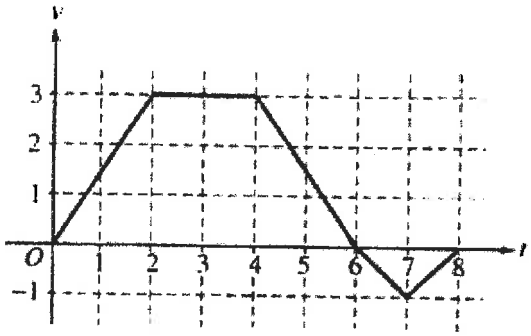
(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$




A bug begins to crawl up a vertical wire at time $t = 0$. The velocity $v$ of the bug at time $t$, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of $t$ does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

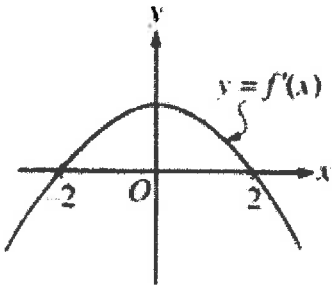
(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

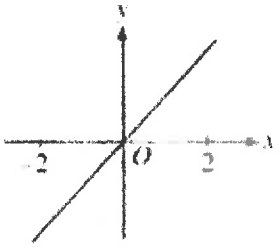
(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

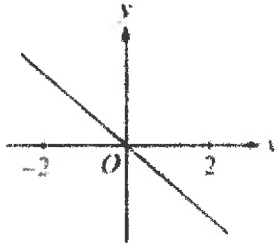



11. The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$?

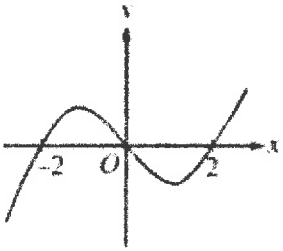
(A)



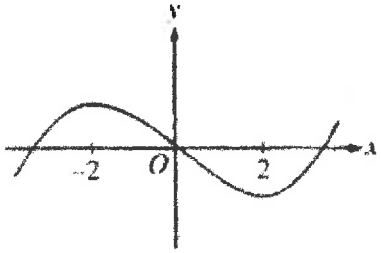

(B)



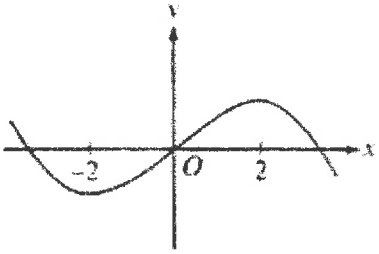

(C)




(D)




(E)




12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

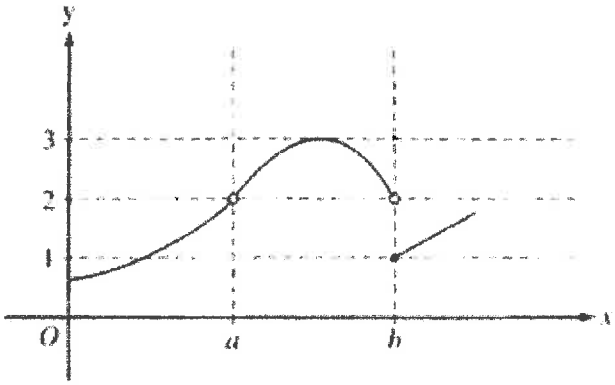
(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2,2)$

13. Let $f$ be a function defined for all real numbers $x$. If $f'(x) = \dfrac{|4-x^2|}{x-2}$, then $f$ is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

14. Let $f$ be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of $f$ at $x = 3$ is used to find an approximation to a zero of $f$, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5




15. The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\frac{\sin 5 - \sin 3}{8}$

(B) $\frac{\sin 5 - \sin 3}{2}$

(C) $\frac{\sin 3 - \sin 5}{2}$

(D) $\frac{\sin 3 + \sin 5}{2}$

(E) $\frac{\sin 3 + \sin 5}{8}$

21. $\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $e$ (E) nonexistent

22. What are all values of $x$ for which the function $f$ defined by $f(x)=(x^2-3)e^{-x}$ is increasing?

(A) There are no such values of $x$.

(B) $x<-1$ and $x>3$

(C) $-3<x<1$

(D) $-1<x<3$

(E) All values of $x$

79. Let $f$ be a function such that $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=5$. Which of the following must be true?

I. $f$ is continuous at $x=2$.

II. $f$ is differentiable at $x=2$.

III. The derivative of $f$ is continuous at $x=2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

3. The function $f$ given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) $-1$ (B) $-\frac{\sqrt{5}}{5}$ (C) $0$ (D) $\frac{\sqrt{5}}{5}$ (E) $1$

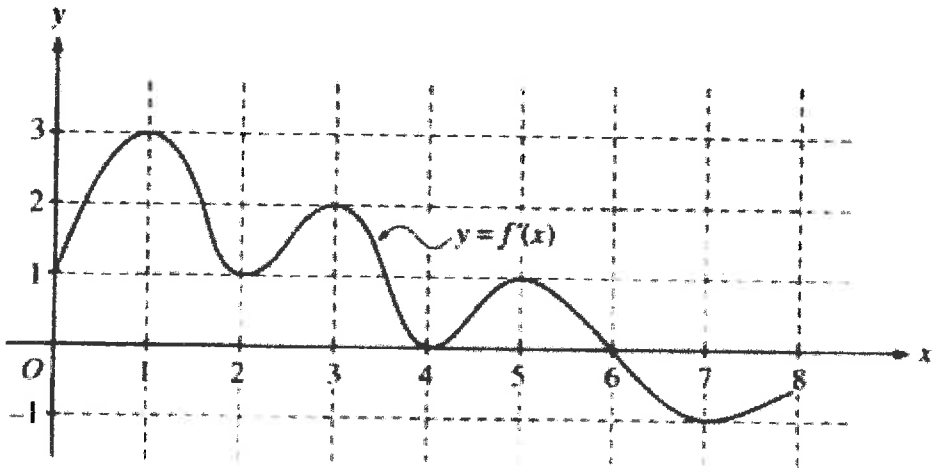
4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) $x^3$ (E) $x^2+x^3$

5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) $1$ (B) $\frac{3}{2}$ (C) $2$ (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

Questions 7-9 refer to the graph and the information below.




The function $f$ is defined on the closed interval $[0,8]$. The graph of its derivative $f'$ is shown above.

7. The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of $f$ at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

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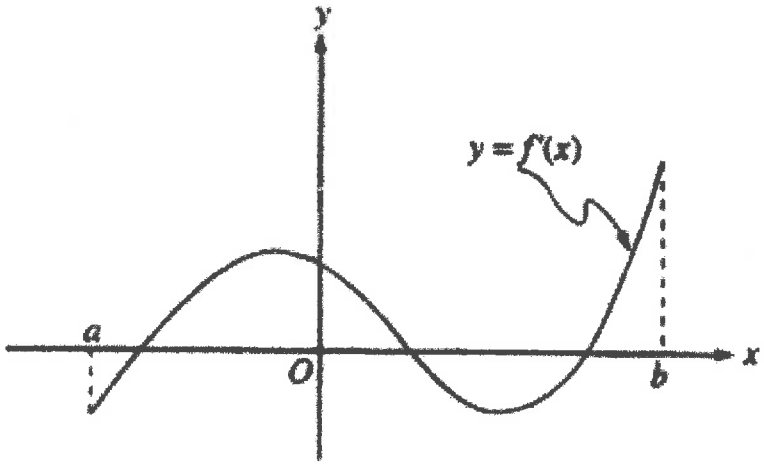
8. How many points of inflection does the graph of $f$ have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of $x$ does the absolute minimum of $f$ occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8




12. The graph of $f'$, the derivative of $f$, is shown in the figure above. Which of the following describes all relative extrema of $f$ on the open interval $(a,b)$?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

16. $\lim_{h\to 0} \dfrac{e^h - 1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $e$ (E) nonexistent

6. The line normal to the curve $y = \sqrt{16 - x}$ at the point $(0,4)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) $-8$