AP Calculus AB – Review Assignments

Date	Section	Assignment DUE
Mar. 3	**Test Chapter 7**	None
Mar. 7	FRQ 2005 / Exam Info	Review Day 1
Mar. 9	FRQ 2006	Review Day 2
Mar. 11	MC 2008	Review Day 3
Mar. 15	Calculator Exercises	Review Day 4
Mar. 17	FRQ 2007	Review Day 5
Mar. 21	FRQ 2008	Review Day 6
Mar. 23 (LS)	Questions on Review Day 1-6, Quiz	None
Mar. 25	FRQ 2009	Review Day 7
Mar. 29	FRQ 2010	Review Day 8
Mar. 31	Questions on Review Day 7-9, Quiz	Review Day 9
Apr. 4	Practice Final MC	None
Apr. 7	Practice Final FRQ	Multiple Choice Take Home Test
BREAK		
Apr. 25	**Optional** Review Session 12:00 noon – 5:00 pm • Grade Practice Final • Questions on Review Day 10,11 • Go over Multiple Choice Packet	
Apr. 26	**Final Exam Part 1**	Review Day 10 Review Day 11
Apr. 28	**Final Exam Part 2**	
May 2	Final Exam Results	
May 4	**AP CALC AB EXAM**	

Posted Documents on VMHS Website

- Calculus Summary (5 page)
- Calculus Summary (1 page)
- College Board Course Overview (60 pages)
  - Syllabus, Practice Problems, Everything you need to know about the course
- Multiple Choice Take Home Test (102 problems) (16 pages)
  - Must show all work
- 2008 Multiple Choice Packet
- Practice Exam
- Review Assignment Packet

Day 1

AP Calc AB Review

Determine the limits of the following

1. $\lim_{x\to 2}\frac{x-2}{x-2}$

2. $\lim_{x\to 5}(2x^2-4x+7)$

3. $\lim_{x\to 2}\frac{x^2+x-6}{x-2}$

4. $\lim_{x\to -1}\frac{x^3+1}{x+1}$

5. $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$

6. $\lim_{x\to 0}\frac{\sin x}{x}$

7. $\lim_{x\to \pi/2}\frac{\cos x}{x-\pi/2}$

8. $\lim_{x\to \infty}\frac{x^2+4}{x-x^2}$

9. $\lim_{x\to 2}\frac{x}{4-x^2}$

10. $\lim_{x\to -\infty}\frac{2x+3}{1-x^2}$

14. Let $f(x)=\begin{cases} x^2-2, & x<1 \\ -\frac{1}{2}x+1, & x\ge 1\end{cases}$

Find
(a) $\lim_{x\to 1^+} f(x)$ (b) $\lim_{x\to 1^-} f(x)$ (c) $\lim_{x\to 1} f(x)$

16. Let $y=f(x)$ be the function shown at right. Which of the following statements is false?

(A) $\lim_{x\to 1} f(x)=1$
(B) $\lim_{x\to 2^-} f(x)=2$
(C) $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)$
(D) $\lim_{x\to -1} f(x)=2$
(E) $\lim_{x\to -1^+} f(x)=2$

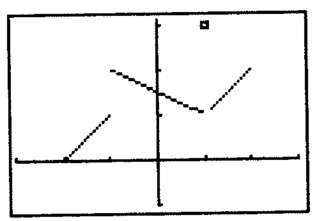


Figure 16

17. The graph of function *f* is shown at right. At which of the following points is *f* continuous?

(A) x = -3
(B) x = -1
(C) x = 1
(D) x= 3
(E) all of the above

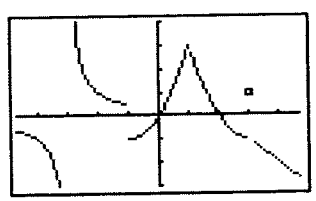
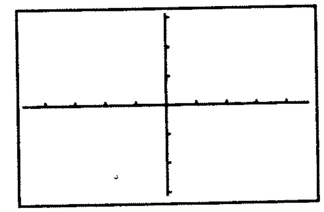


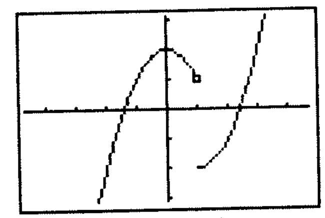
Figure 17

18. The function *f* shown in problem 17 has a removable discontinuity at which of the following points?
(A) x = -3 (B) x = -1 (C) x = 1 (D) x = 3 (E) x = 0

19. Sketch the graph of a function *f* that has the following properties.
i *f(-2)* exists
ii $\lim_{x\to -2} f(x)$ exists
iii *f* is not continuous at x = -2
iv $\lim_{x\to 1} f(x)$ does not exist



20. Let *g(x)* be the function as shown at right. Use the graph to answer the following
(a) $\lim_{x\to 1^-} g(x)$
(b) $\lim_{x\to 1^+} g(x)$
(c) $\lim_{x\to 1} g(x)$
(d) *g(1)*



Graph of g(x)

26. Let *f* be the function defined by $f(x)=2xe^{2x}$
(a) Find $\lim_{x\to -\infty} f(x)$ and $\lim_{x\to \infty} f(x)$
(b) Find the absolute minimum value of *f*. Justify your answer.
(c) What is the range of *f*?

Day 2

Differentiate the following

1) $f(x) = x^{14} + 10x^2 + 7x - 4$

2) $y = x^{-2} - 3x^{-5} + 3x^{-8}$

3) $g(x) = \dfrac{x^3 - 3x + 5}{x^2}$

4) $f(x) = \cos\left(\dfrac{x-1}{x+1}\right)$

5) $h(x) = \dfrac{5x - 2e^x}{4x+3}$

6) $y = (x^2 + 4x - 3)^6 (x^3 - 2x^2 - 6x)^4$

7) $k(x) = \dfrac{(3x-2)^6}{(2x+1)^7}$

8) $f(x) = \ln(x^{3/2} + 2x^{3/5} - 4x^{4/7})$

9) $f(x) = \dfrac{1}{\sqrt[4]{x^2+2x+3}}$

10) $g(x) = \sqrt[3]{\dfrac{3x+4}{2x+3}}$

11) $F(x) = \arctan(4x + 3)$

12) $f(x) = \sin^3(3x^2)\, e^{-4x}$

13) $f(x) = \cot 3x \sec^4 2x$

14) $y = (\sin x)^{\tan x}$

Implicit derivatives

15) $x^2 - \ln(2xy) + 3y^2 = 2$

16) $y^3 + 3\cos x\, y^2 + 3x^3 - 5 = 0$

17) Write the equation of the tangent and normal lines to the graph of $4x^2 - 9y^2 = 36$ when $x = 6$.

1998 AB 4

Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$

(a) Find the slope of the graph of f at the point where $x = 1$

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$

(c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$

Day 3

1992 AB4

Consider the curve defined by $y + \cos y = x + 1$ for $0 \le y \le 2\pi$

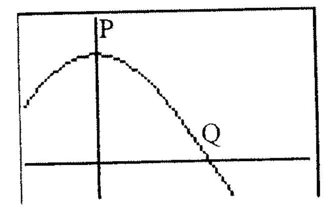
(a) Find $\dfrac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

1996 AB2

Let f be the function given by $f(x) = 3\cos x$. As shown in the figure at the right, the graph of f crosses the y-axis at P (0, 3) and the x-axis at $Q(\pi/2, 0)$.



(a) Write an equation for the line passing through points P and Q.

(b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis which leads to your conclusion.

(c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to the line PQ.

(d) Let R be the region in the first quadrant bounded by the graph of f and the segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.

1) Given: $\mathbf{2x + xy = 6}$. Write the equation in standard form for the tangent and normal lines at $x = 1$.

2) If $\mathbf{y = \cos x}$, write the equation in slope intercept form of the normal line at $x = \pi/3$.

3) Write the equation of the tangent line to $\mathbf{2x^2 + y^2 - xy = 9}$ when $x = 0$.

4) Write the equation of the line tangent to $\mathbf{f(x) = e^{-x}}$ at $x = 0$.
a) Find the x- and y-intercepts of this tangent line.
b) Use the tangent line to approximate f(0.2)

5) Write the equation of the tangent line and the normal line to $\mathbf{y = 3\sin^{-1}x}$ when $x = 1/2$.

6) Given: $\mathbf{f(x) = x^3 - 5x^2 + 3x + 6}$. Find the coordinates of all relative extrema. Find the x-coordinates of any points of inflection. Sketch the graph.

7) Given: $\mathbf{y = x^4 - 4x^3}$. Find the coordinates of all relative maxima and minima and points of inflection. Discuss the concavity. Find the roots of the function. Sketch the graph.

Day 4

8) A particle moves along the x-axis according to the law **$s = 2t^3 - 9t^2 + 12t - 4$**.

a) For what values of t is **s** increasing?
b) For what values of t is the velocity increasing?
c) Find the velocity when t = 3/2.
d) Find the acceleration when t = 2.

9. Let **$f(x) = 12x^{2/3} - 4x$**. (No calculator)
a) Find the coordinates ~~for~~ all critical points.
b) Determine whether the critical points found in part a) are extrema. Justify your answer.
c) Find the coordinates of all points of inflection.
d) For what values of x is f concave up.
e) Sketch the graph of f.

Let *f* be the function given by $f(x) = x^3 - 5x^2 + 3x + k$ where *k* is a constant.

(A) On what intervals is *f* increasing? Justify your answer.

(B) On what intervals is *f* concave downward? Justify your answer.

(C) Find the value of *k* for which *f* has 11 as its relative minimum.

1996 AB4 BC4

This problem deals with functions defined by $f(x) = x + b\sin x$, where *b* is a positive constant and $-2\pi \le x \le 2\pi$.

a) Sketch the graph of these two functions, $y = x + \sin x$ and $y = x + 3\sin x$
b) Find the *x*-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x$.
c) Are the points of tangency described in part b) relative maximum points of *f*? Why?
d) For all values of $b > 0$, show that all inflection points of the graph of *f* lie on the line $y = x$.

1997 AB4

Let *f* be the function given by $f(x) = x^3 - 6x^2 + p$, where *p* is an arbitrary constant.

a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of *f* in terms of *p*. Show the analysis that leads to our conclusion.
b) For what values of *p* does *f* have three distinct reals roots?
c) Find the value of *p* such that the average value of *f* over the closed interval [-1, 2] is 1.

Day 5

1999 AB4

Suppose that the function *f* has a continuous second derivative for all *x*, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let *g* be the function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all *x*.

a) Write an equation of the line tangent to the graph of *f* at the point where $x = 0$.
b) Is there sufficient information to determine whether or not the grpah of *f* has a point of inflection when $x = 0$? Explain your answer.
c) Given that $g(0) = 4$, write the equation of the line tangent to the graph of *g* at the point where $x = 0$.
d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does *g* have a local maximum at $x = 0$? Justify your answer.

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3) Sand falling from a chute forms a conical pile whose altitude is always equal to 4/3 the radius of the base.
a) How fast is the volume increasing when the radius of the base is 3 feet and is increasing at the rate of 3 inches/minute?
b) How fast is the radius increasing when it is 6 feet and the volume is increasing at the rate of 24 cubic feet per minute?

4) Two parallel sides of a rectangle are being lengthened at the rate of 2 cm/sec, while the other two sides are being shortened in such a way that the figure remains a rectangle with constant area of 50 sq cm. What is the rate of change of the perimeter when the length of an increasing side is
a) 5 cm
b) 10 cm
c) What are the dimensions when the perimeter ceases to decrease?

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6) A barge whose deck is 10 ft below the level of a dock is being drawn in by means of a cable attached to the deck and passing through the ring on the dock. The barge is approaching the dock at 3/4 feet per second. How fast is the cable being pulled in when the boat is 24 ft from the dock?

7) Ship A is sailing due south at 16 km/hr and ship B, 32 km south of ship A, is sailing due east at 12 km/hr.
a) At what rate are they approaching or separating after 1 hour?
b) At what rate are they approaching or separating after 2 hours?
c) When do they cease to approach each other and how far apart are they at that time?

8) A floodlight is on the ground 45 meters from a building. A person 2 meters tall runs from the floodlight directly towards the building at the rate of 5 meters/second. How rapidly is the length of his shadow on the building changing when he is 15 meters from the building?

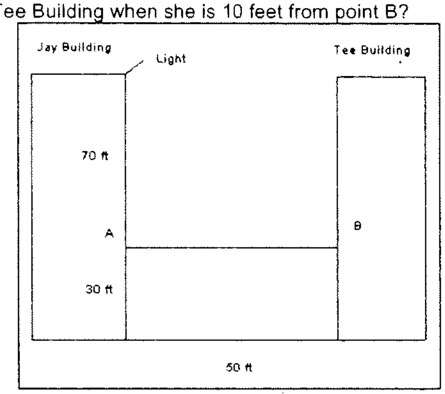
Day 6

10) A water tank is being drained for cleaning. If $G(t)$ represents the number of gallons of water in the tank after t minutes, and $G(t) = 20(30 - t)^2$, find the following.

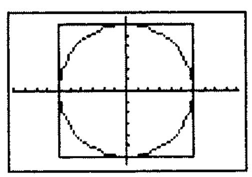
a) How fast is the water draining at $t = 10$ minutes?
b) Find t_0 if the average rate at which the water is draining between t_0 and $2t_0$ is 60 gallons per minute.

11) 1991 AB6
A tightrope is stretched 30 feet above the ground between the Jay and Tee Buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 ft/sec from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.

a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the two buildings?
b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building?
c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B?



12) 1994 BC2 (no calculator)
A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)



(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the <u>area enclosed between</u> the circle and the square. Indicate units of measure.

13) 1990 AB4 (no calculator)
The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. (The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$).

a) At the time when radius of the sphere is 10 centimeters, what is the rate of increase of its volume?
b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of the cross section through the center of the sphere?
c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

Day 7

2. 1990 BC6
Let f and g be continuous functions with the following properties.

(i) $g(x) = A - f(x)$ where A is a constant.
(ii) $\int_1^2 f(x)\,dx = \int_2^3 g(x)\,dx$
(iii) $\int_2^3 f(x)\,dx = -3A$

(a) Find $\int_1^3 f(x)\,dx$ in terms of A.
(b) Find the average value of $g(x)$ in terms of A, over the interval [1, 3].
(c) Find the value of k if $\int_0^1 f(x+1)\,dx = kA$.

3. 1992 AB6
At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$).

a) Find the radius as a function of t.
b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$.

5. 1997 AB6
Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with the initial condition $v(0) = -50$.

a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.
b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.
c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

6. 1998 AB4
Let f be a function with $f(1) = 4$ such that for all points (x,y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$.
b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.
d) Use your solution from part c) to find $f(1.2)$.

Day 8

1981 AB6 BC4

A particle moves along the x-axis so that at time t its position is given by

$x(t) = \sin(\pi t^2)$ for $-1 \le t \le 1$.

a) Find the velocity at time t.
b) Find the acceleration at time t.
c) For what values of t does the particle change direction?
d) Find all values of t for which the particle is moving left.

1987 AB1

A particle moves along the x-axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$ and it's position at time $t = 1$ is $x(1) = 20$.

a) Write an expression for the velocity of the particle $v(t)$ at any time t.
b) For what values of t is the particle at rest?
c) Write an expression for the position $x(t)$ of the particle at any time t.
d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

1989 AB3

A particle moves along the x-axis in such a way that its acceleration at time t for $t \ge 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$ the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

a) Write an equation for the velocity $v(t)$ of the particle.
b) Write an equation for the position $x(t)$ of the particle.
c) For what values of t, $0 \le t \le \pi$ is the particle at rest?

1993 AB2

A particle moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = 2te^{-t}$.

a) Find the acceleration of the particle at $t = 0$.
b) Find the velocity of the particle when its acceleration is 0.
c) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

1999 AB1

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \ge 0$.

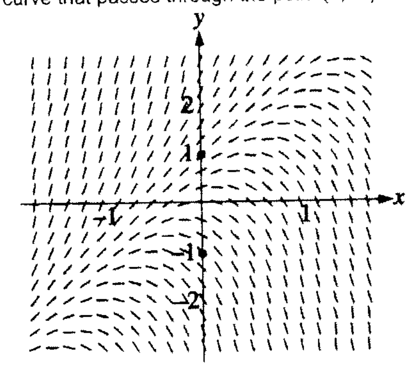
a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
c) Give that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

Day 9

3. 2002 BC5

Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is given below. Sketch the solution curve that passes through the point (0, 1) and sketch the solution curve that passes through the point (0, -1).



(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point (0, 0). If so, is the point a local maximum or a local minimum. Justify your answer.

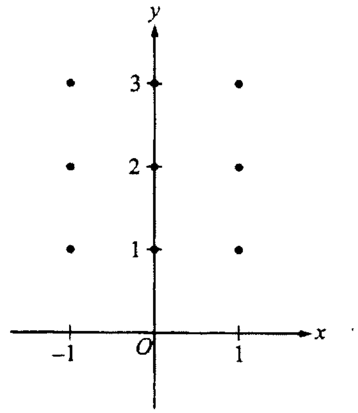
(cont.)

Day 9 (cont.)

1. 1998 BC4

Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated.



(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

Use the Trapezoidal rule to approximate the value of each definite integral for the given n

1. $\int_0^2 x^2\,dx$ n = 5

2. $\int_2^5 \left(\frac{x^3}{\ln x}\right)dx$ n = 6

3. $\int_0^3 x\sin x\,dx$ n = 4

4. The velocity of a car at certain times is given in the table below

Time (minutes)	0	10	20	30	40	50	60	70	80
Velocity (mph)	0	30	60	45	40	55	70	35	60

a. Use a Left hand Riemann Sum with 8 equal subdivisions to estimate the distance traveled by the car over 80 minutes.

b. Use a Mid-point Riemann Sum with 4 equal subdivisions to estimate the distance traveled by the car over 80 minutes.

c. Use a Right hand Riemann Sum with 2 equal subdivisions to estimate the distance traveled by the car over 80 minutes.

Day 10

6. 1999 AB-3, BC-3

t (hours)	0	3	6	9	12	15	18	21	24
R(t) (gallons per hour)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function *R* of time *t*. The table above shows the rate as measured every 3 hours for a 24-hour period.

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

b) Is there some time *t*, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

c) The rate of water flow *R(t)* can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use *Q(t)* to approximate the average rate of water flow during the 24 hour period. Indicate units of measure.

7. 1994 AB6

Let $F(x) = \int_0^x \sin(t^2)\,dt$ for $0 \le x \le 3$

a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0,1] to approximate *F(1)*.

b) On what intervals is *F* increasing?

c) If the average rate of change of *F* is on the closed interval [1,3] is *k*, find $\int_1^3 \sin(t^2)\,dt$ in terms of *k*.

8. 1996 AB3 BC3

The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where *S* is measured in billions of gallons per year and *t* is measured in years from the beginning of 1980.

a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find *C* and *k*.

b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicated units of measure.

c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$

d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

Day 11

1996 AB2

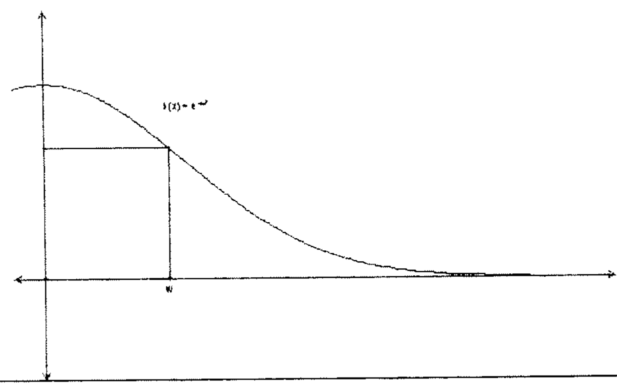
Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

a) Find the area of R.
b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?
c) Find the volume of the solid whose base is the given region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

1996 BC1

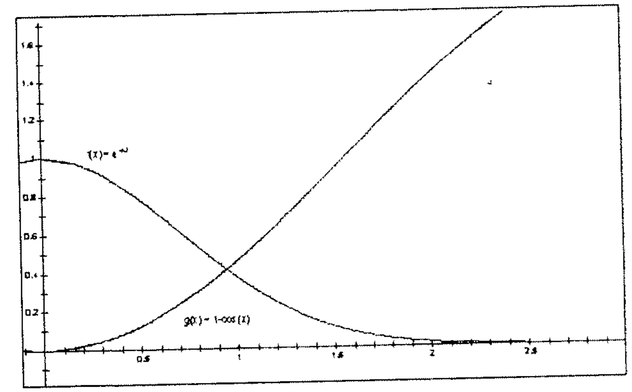
Consider the graph of a function h given by $h(x) = e^{-x^2}$ for $0 \le x \le \infty$.

a) Let R be the unbounded region in the first quadrant below the graph of h. Find the volume of the solid generated when R is revolved about the y-axis.
b) Let $A(w)$ be the area of the shaded rectangle shown in the figure. Show that $A(w)$ has a maximum value when w is the x-coordinate of the point of inflection of the graph of h.



2000 AB1 BC1

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis as shown in the figure.



a) Find the area of region R.
b) Find the volume of the solid generated when the region R is revolved about the x-axis.
c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

1989 BC2 (no calculator)

Let R be the region enclosed by the graph of $y = \frac{x^2}{x^2+1}$, the line $x = 1$, and the x-axis.

a) Find the area of R.
b) Find the volume of the solid generated when R is revolved about the y-axis.