AP CALCULUS
ANTIDERIVATIVES & INITIAL VALUE PROBLEMS PRACTICE

Name Answer Key

1) $\int \pi^3\,dx =$ $\quad \pi^3 x + C$

A) $3\pi^2 x + c$
B) 0
C) $\pi^3 x + c$
D) $3\pi^2 + c$
E) $\frac{\pi^4}{4} + c$

2) $\int (x^4 - x^3 + x^2)\,dx =$ $\quad \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} + C$

~~A)~~ $\frac{x^5}{4} - \frac{x^4}{3} + \frac{x^3}{2} + c$
B) $5x^5 - 4x^4 + 3x^3 + c$
C) $\frac{x^5}{5} - 3x^2 + \frac{x^3}{3} + c$
D) $4x^3 - 3x^2 + 2x + c$
E) $\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} + c$

3) $\int (x^2 + 2)(1 - x)\,dx =$ $\quad \int (x^2 - x^3 + 2 - 2x)\,dx$

$\frac{x^3}{3} - \frac{x^4}{4} + 2x - x^2 + C$

A) $\frac{x^3}{3} - 2x^2 + c$
B) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + c$
C) $-3x^2 + 2x - 2 + c$
D) $-\frac{x^4}{4} + \frac{x^3}{3} - x^2 + 2x + c$
E) $\left(\frac{x^3}{3} - 2x\right)(x - x^2) + c$

4) $\int (x^2 - 2)^2\,dx =$ $\quad \int (x^4 - 4x^2 + 4)\,dx$

$\frac{x^5}{5} - \frac{4x^3}{3} + 4x + C$

A) $\left(\frac{x^2 - 2}{3}\right)^3 + c$
B) $\frac{(x^2 - 2)^3}{6x} + c$
C) $\frac{2x}{3}(x^2 - 2)^3 + c$
D) $\left(\frac{x^3}{3} - 2x\right)^2 + c$
E) $\frac{x^5}{5} - \frac{4x^3}{3} + 4x + c$

5) $\int x^3 (x^3 - 2)\,dx =$ $\quad \int (x^6 - 2x^3)\,dx$

$\frac{x^7}{7} - \frac{x^4}{2} + C$

A) $\frac{x^7}{7} - \frac{x^4}{2} + c$
B) $\frac{x^4}{4}\left(\frac{x^4}{4} - 2x\right) + c$
C) $\frac{3(x^3 - 3)^2}{2} + c$
D) $3x^2\left(\frac{x^4}{4} - 2x\right) + c$
E) $6x^5 - 6x^2 + c$

6) $\int \frac{3x^5 + 2x^3 - x^2}{x^2}\,dx =$ $\quad \int (3x^3 + 2x - 1)\,dx$

$\frac{3x^4}{4} + x^2 - x + C$

A) $\frac{x^6 + x^4 - x^3}{6x^3} + c$
B) $\frac{15x^4 + 6x^2 - 2x}{2x} + c$
C) $18x^6 + 8x^2 - 2x + c$
D) $3x^4 + 2x^2 - x + c$
E) $\frac{3}{4}x^4 + x^2 - x + c$

7) $\int \frac{1}{\sqrt{x}}\,dx =$

$\int x^{-\frac{1}{2}}\,dx$

$2x^{\frac{1}{2}} + C$

$2\sqrt{x} + C$

A) $\frac{1}{2}x\sqrt{x} + c$

B) $\frac{2}{3}x\sqrt{x} + c$

C) $-\frac{1}{2}x\sqrt{x} + c$

D) $2\sqrt{x} + c$

E) $\frac{1}{2}\sqrt{x} + c$

8) $\int \frac{1}{\sqrt[3]{x^2}}\,dx =$

$\int x^{-\frac{2}{3}}\,dx$

$3x^{\frac{1}{3}} + C$

A) $\frac{3}{2}x^{\frac{2}{3}} + c$

B) $-\frac{1}{3}x^{-\frac{1}{3}} + c$

C) $3x^{\frac{1}{3}} + c$

D) $\frac{1}{3}x^{\frac{1}{3}} + c$

E) $-3x^{\frac{1}{3}} + c$

9) $\int\left(\frac{3}{u^{\frac{3}{4}}} - 4\sqrt[3]{u} + 1\right)du =$

$\int(3u^{-\frac{3}{4}} - 4u^{\frac{1}{3}} + 1)\,du$

$12u^{\frac{1}{4}} - 3u^{\frac{4}{3}} + u + C$

A) $\frac{21}{4u^{\frac{7}{4}}} - \frac{1}{3}u^{\frac{4}{3}} + u + c$

B) $\frac{3}{4}u^{\frac{1}{4}} - \frac{16}{3}u^{\frac{4}{3}} + u + c$

C) $12u^{\frac{3}{4}} - 3u^{\frac{5}{3}} + u + c$

D) $\frac{4}{3}u^{\frac{1}{4}} - \frac{3}{16}u^{\frac{4}{3}} + u + c$

E) $12u^{\frac{1}{4}} - 3u^{\frac{4}{3}} + u + c$

10) $\int\left(\sqrt[4]{x^3} - \frac{2}{\sqrt[3]{x^2}}\right)dx =$

$\int(x^{\frac{3}{4}} - 2x^{-\frac{2}{3}})\,dx$

$\frac{4}{7}x^{\frac{7}{4}} - 6x^{\frac{1}{3}} + C$

$\frac{4}{7}\sqrt[4]{x^7} - 6\sqrt[3]{x} + C$

$\frac{4}{7}\sqrt[4]{x^4}\sqrt[4]{x^3} - 6\sqrt[3]{x} + C$

$\frac{4}{7}x\sqrt[4]{x^3} - 6\sqrt[3]{x} + C$

A) $\frac{3}{4\sqrt[4]{x}} - \frac{4}{3\sqrt[3]{x}} + c$

B) $\frac{4}{7}\sqrt[4]{x^3} - \frac{2}{3}\sqrt[3]{x} + c$

C) $\frac{4}{7}x\sqrt[4]{x^3} - 6\sqrt[3]{x} + c$

D) $\frac{4}{7}x\sqrt[4]{x^3} - \frac{6}{5}x\sqrt[3]{x^2} + c$

E) $\frac{7}{4}x\sqrt[4]{x^3} - \frac{5}{6}x\sqrt[3]{x^2} + c$

11) If $g'(x) = 4x^3 + 3x^2 + 6x$ and $g(1) = -3$, then $g(x) =$

A) $x^4 + x^3 + 6x^2 - 11$

B) $\frac{x^4}{4} + \frac{x^3}{3} + 3x^2 - \frac{79}{12}$

C) $12x^2 + 6x + 6$

D) $x^4 + x^3 + 3x^2$

E) $x^4 + x^3 + 3x^2 - 8$

$g(x) = \int(4x^3 + 3x^2 + 6x)$

$g(x) = x^4 + x^3 + 3x^2 + C$

$-3 = 1 + 1 + 3 + C$

$-3 = 5 + C$

$C = -8$

$g(x) = x^4 + x^3 + 3x^2 - 8$

12) Which of the following defines a function f such that $f'(x) = \sqrt{x}$ and the graph of function f pass through the point (9,0)?

A) $f(x) = \frac{2}{3}x\sqrt{x} - 18$

B) $f(x) = x\sqrt{x} - 3x$

C) $f(x) = \frac{x\sqrt{x}}{3} + 9$

D) $f(x) = \frac{1}{2}\sqrt{x} - 3$

E) $f(x) = \frac{3}{2}x\sqrt{x} - 18$

$f(x) = \int\sqrt{x}\,dx$

$f(x) = \int x^{\frac{1}{2}}\,dx$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} + C$

$0 = \frac{2}{3}(9)^{\frac{3}{2}} + C$

$0 = \frac{2}{3}(27) + C$

$0 = 18 + C$

$C = -18$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} - 18$

$f(x) = \frac{2}{3}\sqrt{x^3} - 18$

$\sqrt{x^2} \quad \sqrt{x}$

$f(x) = \frac{2}{3}x\sqrt{x} - 18$

deriv!

13) The slope of a curve at each point (x,y) is given by $4x - 1$. Which of the following is an equation for this curve if it passes through the point (-2,3)?

A) $y = 2x^2 - x - 7$
B) $y = 4x^2 - x - 15$
C) $y = 2x^2 - x + 7$
D) $y = x^2 - 4x - 9$
E) $y = 2x^2 - x$

$y' = 4x - 1$

$y = \int (4x-1)\,dx$

$y = 2x^2 - x + C$

$3 = 2(4) + 2 + C$

$3 = 10 + C$

$C = -7$

$y = 2x^2 - x - 7$

14) If function f has a derivative defined by $f'(x) = \frac{x+1}{\sqrt{x}}$ and $f(1) = 0$, then $f(4) =$

A) $\frac{20}{3}$
B) $-\frac{4}{3}$
C) $\frac{4}{3}$
D) $-\frac{8}{3}$
E) $\frac{3}{4}$

$f(x) = \int \frac{x+1}{\sqrt{x}}\,dx$

$f(x) = \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}})\,dx$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

$0 = \frac{2}{3} + 2 + C$

$0 = \frac{2}{3} + \frac{6}{3} + C$

$0 = \frac{8}{3} + C$

$C = -\frac{8}{3}$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - \frac{8}{3}$

$f(4) = \frac{2}{3}(4)^{\frac{3}{2}} + 2(2) - \frac{8}{3}$

$f(4) = \frac{2}{3}(8) + 4 - \frac{8}{3}$

$f(4) = \frac{8}{3} + 4 = \frac{20}{3}$

15) The slope of the line tangent to the graph of a function f at any point (x,y) is given by $x^3 - x$. If the graph of function f passes through the point (2,1), find f(0).

y'

A) 1
B) 2
C) 3
D) -1
E) 0

$y' = x^3 - x$

$y = \int (x^3 - x)\,dx$

$y = \frac{x^4}{4} - \frac{x^2}{2} + C$

$1 = 4 - 2 + C$

$1 = 2 + C$

$C = -1$

$y = \frac{x^4}{4} - \frac{x^2}{2} - 1$

$y|_{x=0} = 0 - 0 - 1$

$f(0) = -1$

16) A function f has a derivative $f'(x) = 3 - 2x$. An equation of the line tangent to the graph of function f at $x = 2$ is $y - 7 = -(x - 2)$. What is an equation of function f?

point on $f(x)$: $(2,7)$

A) $f(x) = -x^2 + 3x + 5$
B) $f(x) = -3x^2 + x - 3$
C) $f(x) = -x^2 + 3x - 3$
D) $f(x) = 3x^2 + 3x - 1$
E) $f(x) = x^2 - 3x + 3$

$f(x) = \int (3 - 2x)\,dx$

$f(x) = 3x - x^2 + C$

$7 = 6 - 4 + C$

$7 = 2 + C$

$C = 5$

$f(x) = 3x - x^2 + 5$

17) If $h''(x) = x - 2$, $h'(4) = 0$, and $h(0) = 4$, then $h(x) =$

A) $\frac{x^3}{6} - x^2 + 4$
B) $2x^3 - x^2 + 4$
C) $\frac{x^3}{3} - \frac{x^2}{2} + 4$
D) $\frac{x^3}{6} - x^2$
E) $\frac{1}{2}x^2 + 2x + 4$

$h'(x) = \int (x-2)\,dx$

$h'(x) = \frac{x^2}{2} - 2x + C$

$0 = 8 - 8 + C$

$C = 0$

$h'(x) = \frac{x^2}{2} - 2x$

$h(x) = \int (\frac{1}{2}x^2 - 2x)\,dx$

$h(x) = \frac{1}{6}x^3 - x^2 + C$

$4 = 0 - 0 + C \qquad C = 4$

$h(x) = \frac{1}{6}x^3 - x^2 + 4$

18) At each point (x,y) on a curve, $\frac{d^2y}{dx^2} = 6x$. Additionally, the line $y = 6x + 4$ is tangent to the curve at $x = -2$. Which of the following is an equation of the curve that satisfies these conditions?

slope of tang. li

$f'(-2) = 6$

$f(-2) = -8$

$y = 6(-2) + 4$

$y = -12 + 4$

$y = -8$

A) $y = 6x^2 - 32$
B) $y = x^3 - 6x - 12$
C) $y = 2x^3 - 3x$
D) $y = x^3 - 6x + 12$
E) $y = 2x^3 + 3x - 12$

$f'(x) = \int 6x\,dx$

$f'(x) = 3x^2 + C$

$6 = 12 + C$

$C = -6$

$f'(x) = 3x^2 - 6$

$f(x) = \int (3x^2 - 6)\,dx$

$f(x) = x^3 - 6x + C$

$-8 = -8 + 12 + C$

$-8 = 4 + C$

$C = -12$

$f(x) = x^3 - 6x - 12$