

ABSOLUTE VALUE AND PIECEWISE FUNCTIONS

In order to remove the absolute value sign from a function you must:

1. Find the zeroes of the expression inside of the absolute value.
2. Make sign chart of the expression inside the absolute value.
3. Rewrite the equation without the absolute value as a piecewise function. For each interval where the expression is positive we can write that interval by just dropping the absolute value. For each interval that is negative we must take the opposite sign.

Example: Rewrite the following equation without using absolute value.

$$f(x) = |2x + 4|$$

Solution:

$$2x+4=0$$

$$2x=-4$$

$$x=-4/2$$

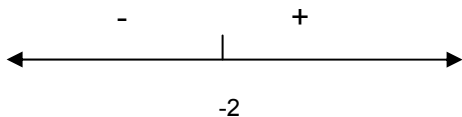
$$x=-2$$

Find where the expression is 0

Subtract 4

Divide by 2

Simplify



Put in any value less than -2 into $2x+4$ and you get a negative.

Put in any value more than -2 into $2x+4$ and you get a positive.

$$f(x) = \begin{cases} -2x-4 & x < -2 \\ 2x+4 & x \geq -2 \end{cases}$$

Write as a piecewise function. Be sure to change the signs of each term for any part of the graph that was negative on the sign chart.

Example: Rewrite the following equation without using absolute value.

$$f(x) = |2x^2 + 5x - 3|$$

Solution:

$$2x^2+5x-3=0$$

$$(2x-1)(x+3)=0$$

$$2x-1=0 \text{ or } x+3=0$$

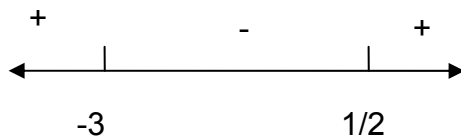
$$x=1/2 \text{ or } x=-3$$

Find where the expression is 0

factor

Set each factor equal to 0

Solve each equation



Put in any value less than -3 into $(2x-1)(x+3)$ and you get a positive number.

Put in any value more than -3 and less than $1/2$ into $(2x-1)(x+3)$ and you get a negative number.

Put in any value more than $1/2$ into $(2x-1)(x+3)$ and you get a positive number.

$$f(x) = \begin{cases} 2x^2 + 5x - 3 & x < -3 \text{ and } x > 1/2 \\ -2x^2 - 5x + 3 & -3 \leq x \leq 1/2 \end{cases}$$

Write as a piecewise function. Be sure to change the signs of each term for any part of the graph that was negative on the sign chart.

Example: Rewrite the following equation without using absolute value.

$$f(x) = |3x - 9| + 2$$

Solution:

$$3x - 9 = 0$$

$$3x = 9$$

$$x = 3$$



$$f(x) = \begin{cases} -3x + 9 + 2 & x < 3 \\ 3x - 9 + 2 & x \geq 3 \end{cases}$$

$$f(x) = \begin{cases} -3x + 11 & x < 3 \\ 3x - 7 & x \geq 3 \end{cases}$$

Find where the expression is 0 (For the part in the absolute value only.)

Add 9

Divide by 3

Put in any value less than 3 into $3x - 9$ and you get a negative.

Put in any value more than 3 into $3x - 9$ and you get a positive.

Write as a piecewise function. Be sure to change the signs of each term that is inside the absolute value for any part of the graph that was negative on the sign chart.

Simplify.

Problem Set VII

Rewrite the following equation without using absolute value. Be sure to show your work, including a sign chart:

1. $f(x) = |-5x + 15|$

2. $f(x) = |(x + 2)(x - 4)|$

3. $f(x) = |7x - 5| + 3$

4. $f(x) = |2x^2 + x - 3|$

5. $f(x) = |5x^2 + 13x - 6| - 2$