Winter Break AP Practice

1. $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

$y = 2x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = 2\left(\dfrac{1}{2}\right)x^{-\frac{1}{2}} - \dfrac{1}{2}\left(\dfrac{1}{2}\right)x^{-\frac{3}{2}}$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}} - \dfrac{1}{4x^{3/2}}$ D.

2. $y = -4\sqrt{2-x}$ max?

$y' = -4\left(\dfrac{1}{2}\right)(2-x)^{-\frac{1}{2}} \cdot (-1)$

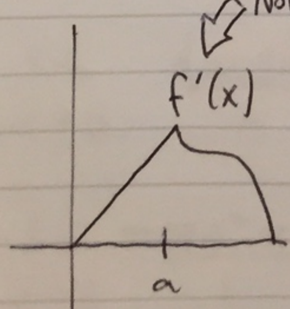
$y' = 2(2-x)^{-\frac{1}{2}}$

$y' = \dfrac{2}{\sqrt{2-x}}$

$0 = \dfrac{2}{\sqrt{2-x}}$ No value of x works, so

E. none of the options

3. Note this is the derivative!



continuous? ✓
differentiable? ✓
max @ x = a? X
poi @ x = a X because of KINK

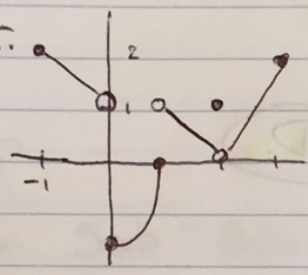
A.

4. $\lim_{x \to 2^-} f(x) = -(2)+2 = 0$

$\lim_{x \to 2} f(x)$ DNE

$\lim_{x \to 2^+} f(x) = 2(2)-2 = 2$

D.

5. 

The function is defined on $[-1, 3]$ if

~~$x \neq 0$~~
~~$x \neq 1$~~
~~$x \neq 2$~~
~~$x \neq 3$~~

These choices would work if any pts. over-lapped.

At each x in $[-1, 3]$

E.

6. Removable discontinuity @

$x = 2$ C.

7. f is continuous on

~~$-1 \leq x \leq 0$~~
$0 < x < 1$
~~$1 \leq x \leq 2$~~
~~$2 \leq x \leq 3$~~

B.

8. jump discontinuity @

$x = 1$

B.

9.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

$$f'(1.5) \approx \frac{f(1.6) - f(1.4)}{1.6 - 1.4}$$

$$\approx \frac{22 - 14}{.2} \approx \frac{8}{.2} \approx 40$$

$$.2\overline{)8.0}^{\,40}$$

D.

10.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6	5	4.4	4.1	4

$$f'(2) = \frac{f(2) - f(1.98)}{2 - 1.98} = \frac{4 - 4.1}{.02} = \frac{-.1}{.02} = -5$$

$$.02\overline{).10}^{\,5}$$

C.

11. $\lim_{x \to 0} \frac{\sin 2x}{x} = \frac{0}{0}$ L'HR!

$$\lim_{x \to 0} \frac{2\cos 2x}{1} = \frac{2\cos(0)}{1} = 2(1) = 2$$

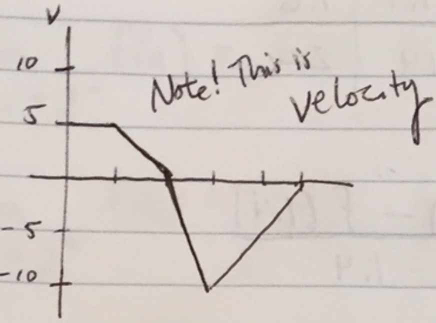
B.

12 $\lim_{x \to 0} \frac{\tan \pi x}{x} = \frac{\tan 0}{0} = \frac{0}{0}$ L'HR!

$$\lim_{x \to 0} \frac{\pi \sec^2 \pi x}{1} = \pi \sec^2(0) = \pi$$

D.

13.



Max ~~speed~~ speed is at

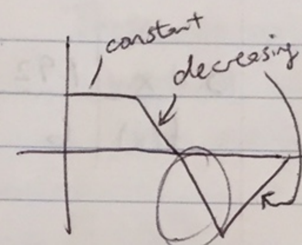
$t = 3$

D Its $10\,m/s$

[The negative implies DIRECTION!]

14. SPEED is increasing from

(2, 3) D



15. Acceleration is positive during (3,5)

E

Acceleration is Velocity's slope

16. Acceleration is undefined @

$x = 1,\ 2,\ 3$ 3 times D.

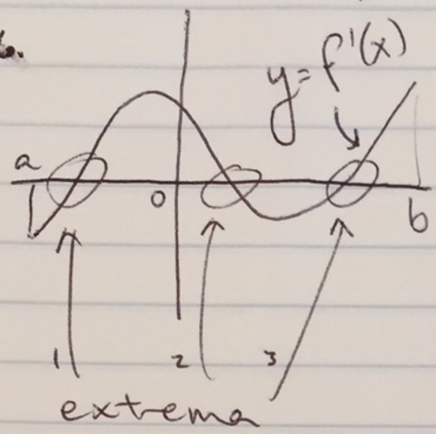
17. $2 < t < 3$, Acceleration is

$$\frac{-10-0}{3-2} = \frac{-10}{1}$$

A.

Note!

6.



A) One rel max & Two rel min

B) Two rel max & One rel min

~~C)~~ Three rel max & One rel min

~~D)~~ One rel max & ~~One~~ Three rel m

~~E)~~ Two rel max & Two rel min

C–E List 4 extrema

	①	②	③		
$f'(x)$	$-$	$+$	$+$	$-$	$+$

↘ ↗ ↘ ↗

MIN MAX MIN

A.

FRQ Practice 1

Solution

(a) Slope at $x = 0$ is $f'(0) = -3$

At $x = 0$, $y = 2$

$y - 2 = -3(x - 0)$

(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.

(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$
\begin{aligned}
g'(0) &= e^{0}(3f(0) + 2f'(0)) \\
&= 3(2) + 2(-3) = 0
\end{aligned}
$$

$y - 4 = 0(x - 0)$

$y = 4$

(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$$
\begin{aligned}
g''(x) &= (-2e^{-2x})(3f(x) + 2f'(x)) \\
&\quad + e^{-2x}(3f'(x) + 2f''(x)) \\
&= e^{-2x}(-6f(x) - f'(x) + 2f''(x))
\end{aligned}
$$

$g''(0) = e^{0}[(-6)(2) - (-3) + 2(0)] = -9$

Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$.

Scoring Guide!

(a) **1**: equation

(b) $\mathbf{2}\begin{cases} 1\text{: answer} \\ 1\text{: explanation} \end{cases}$

(c) $\mathbf{2}\begin{cases} 1\text{: } g'(0) \\ 1\text{: equation} \end{cases}$

(d) $\mathbf{4}\begin{cases} 2\text{: verify derivative} \\ \quad 0/2 \text{ product or chain rule error} \\ \quad <-1> \text{ algebra errors} \\ 1\text{: } g'(0) = 0 \text{ and } g''(0) \\ 1\text{: answer and reasoning} \end{cases}$

FRQ Practice 2

Solution

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

(c) $h'(4) = \dfrac{16-2}{4} = \dfrac{7}{2}$

$$y + 3 = \frac{7}{2}(x-4)$$

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

Scoring Guide!

$$4 : \begin{cases} 1 : x = \pm\sqrt{2} \\ 1 : \text{analysis} \\ 2 : \text{conclusions} \\ \quad < -1 > \text{ not dealing with discontinuity at } 0 \end{cases}$$

$$3 : \begin{cases} 1 : h''(x) \\ 1 : h''(x) > 0 \\ 1 : \text{answer} \end{cases}$$

1 : tangent line equation

1 : answer with reason

CHAIN RULE PRACTICE (IT SAYS *RETAKE* IN THE HEADER)

Answers to Chain Rule ~~Retake~~ Practice (ID: 1)

1) $\frac{dy}{dx} = 5\left((3x^4+4)^3-3\right)^4 \cdot 3(3x^4+4)^2 \cdot 12x^3$

$= 180x^3\left((3x^4+4)^3-3\right)^4 \cdot (3x^4+4)^2$

2) $\frac{dy}{dx} = 3\left((4x^3-3)^5+1\right)^2 \cdot 5(4x^3-3)^4 \cdot 12x^2$

$= 180x^2\left((4x^3-3)^5+1\right)^2 \cdot (4x^3-3)^4$

3) $\frac{dy}{dx} = \frac{1}{3x^2} \cdot 6x$

$= \frac{2}{x}$

4) $\frac{dy}{dx} = e^{3x^5} \cdot 15x^4$

5) $\frac{dy}{dx} = \frac{1}{4x^5} \cdot 20x^4$

$= \frac{5}{x}$

6) $\frac{dy}{dx} = \frac{1}{2x^3} \cdot 6x^2$

$= \frac{3}{x}$

7) $\frac{dy}{dx} = \cos 3x^4 \cdot 12x^3$

$= 12x^3 \cos 3x^4$

8) $\frac{dy}{dx} = \sec 5x^2 \cdot \tan 5x^2 \cdot 10x$

$= 10x \sec 5x^2 \cdot \tan 5x^2$

9) $\frac{dy}{dx} = -\sin 4x^4 \cdot 16x^3$

$= -16x^3 \sin 4x^4$

10) $\frac{dy}{dx} = \sec^2 2x^2 \cdot 4x$

$= 4x \cdot \sec^2 2x^2$

ALGEBRA PRACTICE

Winter Break - Algebra Review

1. $y^4(6-y)(5+y)$

$= y^4(30+6y-5y-y^2)$

$= y^4(30+y-y^2)$

$\boxed{= 30y^4+y^5-y^6}$

2. $(t-5)^2-2(t-3)(8t-1)$

$= t^2-10t+25-2(8t^2-t-24t+3)$

$-25t$

$= t^2-10t+25-16t^2+50t-6$

$\boxed{= -15t^2+40t+19}$

3. $\frac{1}{x+5}+\frac{2}{x-3}$

$= \frac{1}{x+5}\left(\frac{x-3}{x-3}\right)+\frac{2}{x-3}\left(\frac{x+5}{x+5}\right)$

$= \frac{x-3}{(x^2-3x+5x-15)}+\frac{2x+10}{(x^2-3x+5x-15)}$

$= \frac{x-3+2x+10}{x^2+2x-15} = \frac{3x+7}{x^2+2x-15}$

ALGEBRA PRACTICE

4. $\frac{\cancel{9}^{3}b - \cancel{6}^{2}}{\cancel{3}b} = \frac{3b-2}{b}$

5. $\frac{x^2-1}{(x^2-9x+8)} = \frac{(x+1)\cancel{(x-1)}}{(x-8)\cancel{(x-1)}}$

$= \frac{x+1}{x-8}$

4 answers!

6. $5ab - 8abc = ab(5-8c)$

$x^2 - x - 6 = (x-3)(x+2)$

$2x^2 + 7x - 4 = (2x-1)(x+4)$

$8x^2 + 10x + 3 = ((4x+3)(2x+1))$

24, 6, 4, 10

	$4x$	3
$2x$	$8x^2$	$6x$
1	$4x$	3