3.3 The First and Second Derivative Tests

FOR THE FOLLOWING, FIND: A) THE DOMAIN OF EACH FUNCTION, B) THE x-COORDINATE OF THE LOCAL EXTREMA, AND C) THE INTERVALS WHERE THE FUNCTION IS INCREASING AND/OR DECREASING.

701. $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 1$

702. $g(x) = x^3 - 5x^2 - 8x$

703. $h(x) = x + \dfrac{4}{x}$

704. $p(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

705. $h(x) = (2-x)^2(x+3)^3$

706. $m(x) = 3x\sqrt{5-x}$

707. $f(x) = x^{2/3}(x-5)^{-1/3}$

708. $h(x) = \frac{1}{7}x^{7/3} - x^{4/3}$

709. Find the values of a and b so that the function $f(x) = \frac{1}{3}x^3 + ax^2 + bx$ will have a relative extreme at $(3, 1)$.

710. Find the values of a, b, c, and d so that the function $f(x) = ax^3 + bx^2 + cx + d$ will have relative extrema at $(-1, 1)$ and $(-2, 4)$.

IN THE FOLLOWING PROBLEMS, FIND A) THE COORDINATES OF INFLECTION POINTS AND B) THE INTERVALS WHERE THE FUNCTION IS CONCAVE UP AND/OR CONCAVE DOWN.

711. $g(x) = x^3 - 5x$

712. $h(x) = 2x^3 - 3x^2 - 8x + 1$

713. $h(x) = (3x+2)^3$

714. $p(x) = \dfrac{3}{x^2+4}$

715. $f(x) = \begin{cases} x^2 - 3 & x > 3 \\ 15 - x^2 & x \le 3 \end{cases}$

716. $p(x) = \begin{cases} 2x^2 & x \ge 0 \\ -2x^2 & x < 0 \end{cases}$

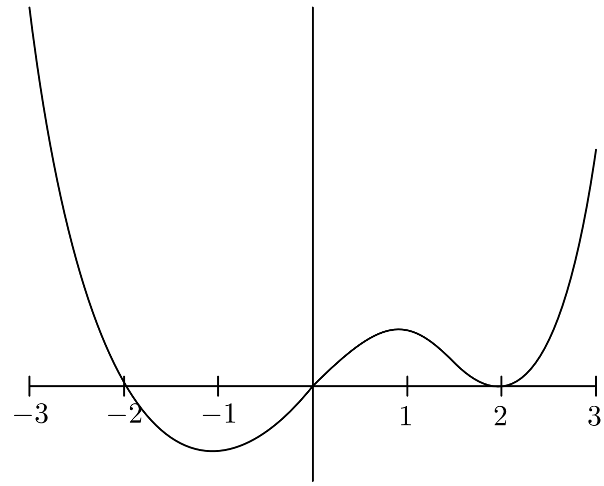
717. Determine the values of a and b so that the function $p(x) = ax^4 + bx^3$ will have a point of inflection at $(-1, 3)$.

718. Determine the values of a, b, and c so that the function $p(x) = ax^3 + bx^2 + cx$ will have an inflection point at $(-1, 3)$ and the slope of the tangent at $(-1, 3)$ will be -2.

The calculus is the greatest aid we have to the application of physical truth in the broadest sense of the word. —*W. F. Osgood*

3.5 Two Derivative Problems

722 (AP). The graph below is the graph of the derivative of a function f.

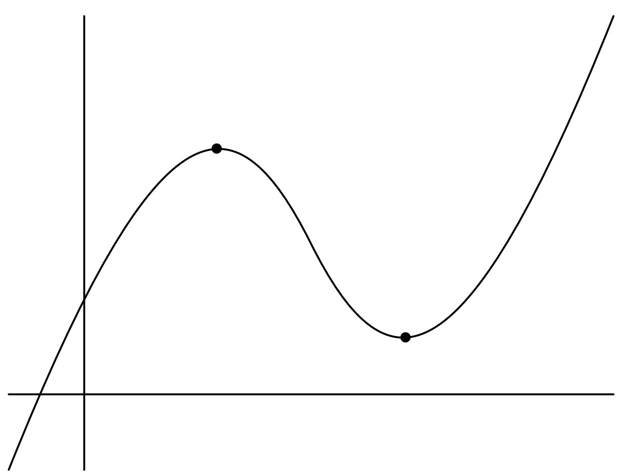


a) Find where f is increasing and where it is decreasing.

b) Find all local maxima and local minima of f.

c) If $f(-3) = -2$, sketch the graph of f.

723 (AP). The graph below is that of a function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are constants. Show that the x-coordinates of the two marked points are given by the formula $x = \dfrac{-b \pm \sqrt{b^2 - 3ac}}{3a}$.



In the fall of 1972 President Nixon announced that the rate of increase of inflation was decreasing. This was the first time a sitting President used the third derivative to advance his case for re-election. —*Hugo Rossi*

585. $-6x^2e^{-2x^3}$

586. $\dfrac{e^x(x-3)}{x^4}$

587. $-2x^{-3}$

588. $6(x-1)10^{3x^2-6x}\ln 10$

589. $3^{2x}2^{3x^2}(\ln 9 + 6x\ln 2)$

590. $\dfrac{2xy+y}{3xy-x}$

591. $\frac{1}{3}(4x+12y-17)$

592. $\dfrac{y}{ye^y+1}$

593. $\dfrac{4\cos(x-3y)}{1+12\cos(x-3y)}$

594. $\dfrac{2}{3\cos y - 2}$

595. $\dfrac{\sin(x-2y)}{2\sin(x-2y)-3}$

598. $\dfrac{-5\csc^2 5x}{2\sqrt{\cot 5x}}$

599. $24\cos 16x$

601. $-6\sin 6x$

602. $e^{\sin x}\cos x$

603. $-3^{\cos x}\ln 3\sin x$

604. $\frac{2}{\ln 3}\cot 2x$

605. $6x$

606. $e^{3x}(\sec^2 x + 3\tan x)$

607. $\dfrac{-2e^{1/x^2}}{x^3}$

608. $\frac{1}{2}xe^{x^2/4}$

610. $e^{\tan x}(1+x\sec^2 x)$

625. $\frac{2}{\sqrt{3}}$

626. $\frac{1}{2}$

627. $\frac{1}{51}$

628. $\frac{1}{2}$

629. $\frac{\sqrt[3]{25}}{2}$

631. $\dfrac{-1}{\sqrt{-x^2+3x-2}}$

634. $\dfrac{-3}{x^2+9}$

635. $\dfrac{1}{|x|\sqrt{x^2-1}}$

636. $\dfrac{-4}{\sqrt{2-4x^2}}$

637. $\dfrac{-1}{\sqrt{2x-x^2}}$

638. $y = ex$

644. (a) 6.7 million ft^3/acre (b) 0.073 and 0.04 million ft^3/acre per year

647. (b) 50 (c) 25 (d) $1-0.04x$ (e) 0

648. (a) $x'(t) = \dfrac{1}{1+t^2}$ is always positive (b) $x''(t) = \dfrac{-2t}{(1+t^2)^2}$ is always negative (c) $\frac{\pi}{2}$

649. (a) $\frac{10}{\sqrt{2}}$ (b) left -10, right 10 (c) when $t=-10$, $v=0$ and $a=10$, when $t=10$, $v=0$, $a=-10$ (d) at $t=-\frac{\pi}{4}$, $v=-10$, speed $=10$, $a=10$

650. (a) $2x$ (b) $2x$ (c) 2 (d) 2 (e) yes

651. (a) $x=-1$ (b) ≈ -1

652. (a) $\mathbb{R}$ (b) $\dfrac{-\cos x}{2\sqrt{1-\sin x}}$ (c) $\{x|x \neq \frac{\pi}{2}+2\pi n, n\in\mathbb{Z}\}$ (d) $y=-\frac{1}{2}x+1$

653. (a) $-\dfrac{2x+y}{x+2y}$ (c) $(6,-3)$ and $(-6,3)$

654. (a) $\frac{24}{\pi}$ in/sec (b) $\frac{120}{\pi}-30$ in^2/sec

655. (a) $\frac{3}{\sqrt{5}}$ m/sec (b) 150 m^2/sec (c) $\frac{3}{125}$ radian/sec

656. (a) $a=0, b=9, c=4$ (b) $x=\pm 2$ (c) $y=0$

658. E

659. D

660. D

661. D

662. E

663. E

664. E

665. B

666. B

667. C

668. D

669. D

670. B

671. D

672. D

673. A

674. D

675. E

681. (a)-(c) no (d) $(\pm 3, 0)$, $(\pm\sqrt{3}, 6\sqrt{3})$, and $(0,0)$

682. $\{x|0<x<5\}$, extreme values are 0 and 144

693. $c=1$

694. $c=\sqrt{\frac{7}{3}}$

695. $\frac{5}{6}$

696. $-1, 0, 1$

698. $0, \frac{1}{5}$

699. No, Rolle's Theorem does not apply since f is not continuous on $[0,1]$.

709. $a=-\frac{19}{9}, b=\frac{11}{3}$

710. $a=6, b=27, c=36, d=16$

711. (a) $(0,0)$ (b) ccup for $x>0$, ccdown for $x<0$

714. (a) $\left(\pm\frac{2}{\sqrt{3}}, \frac{9}{16}\right)$ (b) ccup for $x<-\frac{2}{\sqrt{3}}$ and $x>\frac{2}{\sqrt{3}}$, ccdown for $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$

715. (a) $(3,6)$ (b) ccup for $x>3$, ccdown for $x<3$

717. $a=-3, b=-6$

718. $a=-1, b=-3, c=-5$

722. (b) max at $x=-2$, min at $x=0$