AP Calculus AB – Worksheet 83 **The Second Derivative and The Concavity Test**

For #1-3

a) **Find and classify the critical point(s).**
b) **Find the interval(s) where $f(x)$ is increasing.**
c) **Find the interval(s) where $f(x)$ is decreasing.**

1. $f(x)=x^2-x-1$	2. $f(x)=2x^4-4x^2+1$	3. $f(x)=xe^{1/x}$

For # 4-6

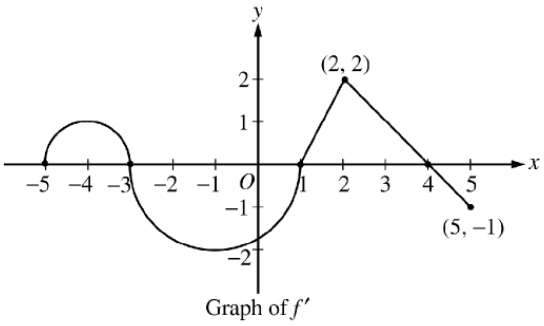
a) **Find the *x*-coordinate of the point(s) of inflection.**
b) **Find the interval(s) where $f(x)$ is concave up.**
c) **Find the interval(s) where $f(x)$ is concave down.**

4. $f(x)=4x^3+21x^2+36x-20$	5. $f(x)=2x^{1/5}+3$	6. $f(x)=-x^4+4x^3-4x+1$

For #7-10, find all points of inflection of the function. Justify your answer.

7. $y=xe^x$	8. $f(x)=\tan^{-1}x$
9. $f(x)=x^{1/3}(x-4)$	10. $y=\dfrac{x^3-2x^2+x-1}{x-2}$

Free Response Question



Graph of f'

Let *f* be a function defined on the closed interval $-5 \le x \le 5$ with $f(1)=3$. The graph of f', the derivative of *f*, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values of *x* at which *f* has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values of *x* at which *f* has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of *f* (not shown) is concave up. Justify your answer.

(d) Find all intervals on which the graph of *f* (not shown) has a positive slope. Justify your answer.

Answers:

	Relative Min f' changes from $-$ to $+$	**Relative Max** f' changes from $+$ to $-$	**Increasing** $f' > 0$	**Decreasing** $f' < 0$
1.	$\left(\frac{1}{2}, -\frac{5}{4}\right)$	None	$\left(\frac{1}{2}, \infty\right)$	$\left(-\infty, \frac{1}{2}\right)$
2.	$(-1,-1)$ and $(1,-1)$	$(0,1)$	$(-1,0)$ and $(1,\infty)$	$(-\infty,-1)$ and $(0,1)$
3	$(1,e)$	None	$(-\infty,0)$ and $(1,\infty)$	$(0,1)$

	***x*-coordinate of point of inflection** f'' changes signs	**Concave Up** $f'' > 0$	**Concave Down** $f'' < 0$
4.	$x = -\frac{7}{4}$	$\left(-\frac{7}{4}, \infty\right)$	$\left(-\infty, -\frac{7}{4}\right)$
5.	$x = 0$	$(-\infty, 0)$	$(0, \infty)$
6.	$x = 0$ and $x = 2$	$(0,2)$	$(-\infty,2)$ and $(2,\infty)$

7. $\left(-2, -\frac{2}{e^2}\right)$	**8.** $(0,0)$	**9.** $(0,0)$ and $\left(-2, 6\sqrt[3]{2}\right)$	**10.** $(1,1)$

Free Response Question:

a) f has a relative maximum at $x = -3$ and $x = 4$ because $f'(x)$ changes signs from positive to negative.

b) f has a point of inflection at $x = -1$ and $x = 2$ because f'' changes signs

c) f is concave up on $(-5,-4)$ and $(-1,2)$ because f' is increasing or $f'' > 0$

d) f has a positive slope on $(-5,-3)$ and $(1,4)$ because $f' > 0$.