

2.8 The RULES: Power Product Quotient Chain

447. Let $f(x) = \begin{cases} 3 - x & x < 1 \\ ax^2 + bx & x \geq 1 \end{cases}$ where a and b are constants.

- a) If the function is continuous for all x , what is the relationship between a and b ?
 b) Find the unique values for a and b that will make f both continuous and differentiable.

448. Suppose that $u(x)$ and $v(x)$ are differentiable functions of x and that

$$u(1) = 2, \quad u'(1) = 0, \quad v(1) = 5, \quad \text{and} \quad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

a) $\frac{d}{dx}(uv)$ b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ d) $\frac{d}{dx}(7v - 2u)$

449. Graph the function $y = \frac{4x}{x^2 + 1}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called *Newton's serpentine*.) Find the tangent lines at the origin and at the point $(1, 2)$.

450. Graph the function $y = \frac{8}{x^2 + 4}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called the *witch of Agnesi*.) Find the tangent line at the point $(2, 1)$.

FIND THE DERIVATIVE OF THE GIVEN FUNCTION. EXPRESS YOUR ANSWER IN SIMPLEST FACTORED FORM.

451. $A(z) = (3z - 5)^4$

460. $h(u) = \sqrt{u-1} \sqrt[3]{2u+3}$

452. $q(u) = (3u^5 - 2u^3 - 3u - \frac{1}{3})^3$

461. $f(x) = \frac{3x}{x+5}$

453. $b(y) = (y^3 - 5)^{-4}$

462. $g(y) = \frac{4y-3}{3-2y}$

454. $c(d) = \sqrt[3]{(5d^2 - 1)^5}$

463. $p(x) = \frac{x^2 + 10x + 25}{x^2 - 10x + 25}$

455. $u(p) = \frac{3p^2 - 5}{p^3 + 2p - 6}$

464. $m(x) = \frac{7x}{1-3x}$

456. $V(x) = \frac{\sqrt{5x^3}}{5x^3}$

465. $f(x) = \frac{3}{x^2} - \frac{x^2}{3}$

457. $f(x) = 3x^{1/3} - 5x^{-1/3}$

466. $g(x) = \left(\frac{4x-3}{5-3x}\right)(2x+7)$

458. $g(z) = \frac{1}{\sqrt{36-z^2}}$

459. $p(t) = (3-2t)^{-1/2}$

467. $F(x) = 10x^{27} - 25x^{1/5} + 12x^{-12} + 350$

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction. —*Leo Tolstoy*

- 372.** $2x - 3 + 5x^{-2} - 14x^{-3}$
373. $12x + 13$
374. $\frac{1}{2}x^{-1/2} + \frac{34}{3}x^{-1/3}$
375. $2\pi x^2 + 20\pi x$
380. $14x, 14, 14x, 28$
381. no
382. no
383. yes
384. no
385. yes
386. yes
392. $(-2, -5)$
394. 6π
396. yes
397. (a) $8(x - 2) = y - 1$ (b) $[-4, \infty)$ (c) $8(x - 2) = y - 1$ and $8(x + 2) = y - 1$
398. no
399. no
400. yes
401. yes
402. no
403. no
404. no
405. no
409. (a) $6t$ (b) -6
410. (b) $-(x + 1) = y + 2$
412. (a) 280 (b) mg/day
428. a, d, and e
429. d.n.e.
430. 0
431. 0
432. $\frac{\pi}{2}$
433. $-\frac{\pi}{2}$
434. $\frac{\pi}{2}$
435. only one is even, only one is neither
436. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$
437. $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$
438. $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$
439. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
440. $\frac{2\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}$
441. π
442. (a) 9000 gal (b) 300 gal/hr (c) yes, the tank's volume is zero at $t = 30$ hrs
444. (a) yes (b)-(d) no
445. none must be true
446. (a) $a = b + 2$ (c) $a = 3, b = 1$
453. $-12y^2(y^3 - 5)^{-5}$
455. $\frac{-3p^4 + 21p^2 - 36p + 10}{(p^3 + 2p - 6)^2}$
456. $\frac{-3}{2x^{5/2}\sqrt{5}}$
458. $\frac{-z}{(36 - z^2)^{3/2}}$
460. $\frac{10u + 5}{6\sqrt{u - 1}(2u + 3)^{2/3}}$
461. $\frac{15}{(x + 5)^2}$
463. $\frac{-20(x + 5)}{(x - 5)^3}$
464. $\frac{7}{(1 - 3x)^2}$
466. $\frac{-24x^2 + 80x + 47}{(5 - 3x)^2}$
483. $-3(x - \frac{\pi}{4}) = y - 4; \frac{\pi}{2}$
485. $\cos x$
486. $\pi(x - 1) = y - 2$
487. $\csc \theta(\cot^2 \theta + \csc^2 \theta)$
488. $\sec \theta(\tan^2 \theta + \sec^2 \theta)$
489. $2 \sin \theta$
490. $-\sin \theta - \cos \theta$
497. $10(x - 1) = y - 2$
499. no
500. no
501. no
502. yes
503. no
504. no
505. (a) 5 (b) 0 (c) 8 (d) 2 (e) 6 (f) -1
517. $y = 1$
518. (a) $\frac{5}{4}(x - 4) = y - 2$ and $\frac{4}{5}(x - 2) = y - 4$ (b) 0 and $3\sqrt[3]{2}$ (c) 0 and $3\sqrt[3]{4}$
519. $(\pm\sqrt{7}, 0)$; slope is -2
520. $(3, -1)$
521. (a) $\frac{3x^y - y^2}{2xy - x^3}$
 (b) at $(1, -2)$ the tangent is $2(x - 1) = y + 2$, at $(1, 3)$ the tangent is $y = 3$
 (c) $\sqrt[5]{-24}$
530. $AC = \frac{5}{4}\sqrt{29}, BC = \frac{25}{2}$
531. $16\sqrt{3}$
532. 50π
533. 15
534. $3\sqrt[3]{9}$
535. $6\sqrt{3}$
536. $2\sqrt{3}$
545. (a) 34994 dollars/week (b) 200 dollars/week (c) 34794 dollars/week
546. $\frac{5}{16}\sqrt{3}$ m/hr
547. 18 m/sec
548. 3 ft/sec
550. $(\frac{1}{4}, \frac{1}{2})$
551. (a) $s = \frac{7}{8}d$ (b) $\frac{35}{8}$ ft/sec
552. 1 ft/min; 40 π ft²/min
553. ≈ 7.1 in/min
554. 12 in³/sec
555. (a) $\pm\frac{5}{3}$ units/sec (b) ± 24
572. $\ln x$
579. $\frac{2^x \ln \frac{2}{5} + \ln 5}{5^x}$
582. $\frac{3}{5(3x - 2)}$
584. $\frac{x \ln x - x + 2}{x(\ln x)^2}$