2.4 Going Off on a Tangent

FOR THE FOLLOWING FIVE PROBLEMS, FIND AN EQUATION FOR THE TANGENT LINE TO THE CURVE AT THE GIVEN x-COORDINATE.

387. $y = 4 - x^2$; $x = -1$

388. $y = 2\sqrt{x}$; $x = 1$

389. $y = x - 2x^2$; $x = 1$

390. $y = x^{-3}$; $x = -2$

391. $y = x^3 + 3x$; $x = 1$

392. At what points does the graph of $y = x^2 + 4x - 1$ have a horizontal tangent?

393. Find an equation for the tangent to the curve $y = \sqrt{x}$ that has slope $\frac{1}{4}$.

394. What is the instantaneous rate of change of the area of a circle when the radius is 3 cm?

395. What is the instantaneous rate of change of the volume of a ball when the radius is 2 cm?

396. Does the graph of $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ have a tangent at the origin? Justify your answer.

397. Consider the curve $y = x^3 - 4x + 1$.

   a) Find an equation for the tangent to the curve at the point $(2, 1)$.

   b) What is the range of values of the curve's slope?

   c) Find equations for the tangents to the curve at the points where the slope of the curve is 8.

DETERMINE WHICH OF THE FOLLOWING FUNCTIONS ARE DIFFERENTIABLE AT $x = 0$.

398. $y = x^{1/3}$

399. $y = x^{2/3}$

400. $y = x^{4/3}$

401. $y = x^{5/3}$

402. $y = x^{1/4}$

403. $y = x^{5/4}$

404. $y = x^{1/5}$

405. $y = x^{2/5}$

406. Based on the answers from the problems above, find a pattern for the differentiability of functions with exponents of the following forms: $x^{\text{even/odd}}$, $x^{\text{odd/odd}}$, $x^{\text{odd/even}}$.

To err is human, but when the eraser wears out ahead of the pencil, you're overdoing it. —*Josh Jenkins*

2.10 Tangents, Normals, and Continuity (Revisited)

491. Find the equation of the tangent line to the curve $y = \sqrt{x^2 - 3}$ at the point $(2, 1)$.

492. Find the equation of the normal line to the curve $y = (3x - 1)^2(x - 1)^3$ at $x = 0$.

493. Find the equation of the tangent line to the curve $y = \sqrt{3x - 1}$ that is perpendicular to the line $3y + 2x = 3$.

494. Find the equation of the normal line to the curve $y = x\sqrt{25 + x^2}$ at $x = 0$.

495. Find the equation of the tangent line to the curve $y = \dfrac{2 - x}{5 + x}$ at $x = 1$.

496. Find the equation of the normal line to the curve $y = \dfrac{5}{(5 - 2x)^2}$ at $x = 0$.

497. Find the equation of the tangent line to the curve $y = 3x^4 - 2x + 1$ that is parallel to the line $y - 10x - 3 = 0$.

498. The point $P(3, -2)$ is not on the graph of $y = x^2 - 7$. Find the equation of each line tangent to $y = x^2 - 7$ that passes through P.

FOR THE FOLLOWING SIX PROBLEMS, DETERMINE IF f IS DIFFERENTIABLE AT $x = a$.

499. $f(x) = |x + 5|$; $a = -5$

500. $f(x) = \begin{cases} x + 3 & x \leq -2 \\ -x - 1 & x > -2 \end{cases}$ $a = -2$

501. $f(x) = \begin{cases} 2 & x < 0 \\ x - 4 & x \geq 0 \end{cases}$ $a = 0$

502. $f(x) = \begin{cases} -2x^2 & x < 0 \\ 2x^2 & x \geq 0 \end{cases}$ $a = 0$

503. $f(x) = \begin{cases} x^2 - 5 & x < 3 \\ 3x - 5 & x \geq 3 \end{cases}$ $a = 3$

504. $f(x) = \begin{cases} \sqrt{2 - x} & x < 2 \\ (2 - x)^2 & x \geq 2 \end{cases}$ $a = 2$

505. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and at $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivatives of the following combinations of f and g at the given value of x.

a) $3f(x) - g(x)$, $x = -1$

b) $[f(x)]^3[g(x)]^3$, $x = 0$

c) $g(f(x))$, $x = -1$

d) $f(g(x))$, $x = -1$

e) $\dfrac{f(x)}{g(x) + 2}$, $x = 0$

f) $g(x + f(x))$, $x = 0$

372. $2x - 3 + 5x^{-2} - 14x^{-3}$

373. $12x + 13$

374. $\frac{1}{2}x^{-1/2} + \frac{34}{3}x^{-1/3}$

375. $2\pi x^2 + 20\pi x$

380. $14x, 14, 14x, 28$

381. no

382. no

383. yes

384. no

385. yes

386. yes

392. $(-2, -5)$

394. 6π

396. yes

397. (a) $8(x - 2) = y - 1$ (b) $[-4, \infty)$ (c) $8(x - 2) = y - 1$ and $8(x + 2) = y - 1$

398. no

399. no

400. yes

401. yes

402. no

403. no

404. no

405. no

409. (a) $6t$ (b) -6

410. (b) $-(x + 1) = y + 2$

412. (a) 280 (b) mg/day

428. a, d, and e

429. d.n.e.

430. 0

431. 0

432. $\frac{\pi}{2}$

433. $-\frac{\pi}{2}$

434. $\frac{\pi}{2}$

435. only one is even, only one is neither

436. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

437. $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

438. $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$

439. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

440. $\frac{2\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}$

441. π

442. (a) 9000 gal (b) 300 gal/hr (c) yes, the tank's volume is zero at $t = 30$ hrs

444. (a) yes (b)-(d) no

445. none must be true

446. (a) $a = b + 2$ (c) $a = 3$, $b = 1$

453. $-12y^2(y^3 - 5)^{-5}$

455. $\dfrac{-3p^4 + 21p^2 - 36p + 10}{(p^3 + 2p - 6)^2}$

456. $\dfrac{-3}{2x^{5/2}\sqrt{5}}$

458. $\dfrac{-z}{(36 - z^2)^{3/2}}$

460. $\dfrac{10u + 5}{6\sqrt{u - 1}(2u + 3)^{2/3}}$

461. $\dfrac{15}{(x + 5)^2}$

463. $\dfrac{-20(x + 5)}{(x - 5)^3}$

464. $\dfrac{7}{(1 - 3x)^2}$

466. $\dfrac{-24x^2 + 80x + 47}{(5 - 3x)^2}$

483. $-3(x - \frac{\pi}{4}) = y - 4; \frac{\pi}{2}$

485. $\cos x$

486. $\pi(x - 1) = y - 2$

487. $\csc\theta(\cot^2\theta + \csc^2\theta)$

488. $\sec\theta(\tan^2\theta + \sec^2\theta)$

489. $2\sin\theta$

490. $-\sin\theta - \cos\theta$

497. $10(x - 1) = y - 2$

499. no

500. no

501. no

502. yes

503. no

504. no

505. (a) 5 (b) 0 (c) 8 (d) 2 (e) 6 (f) -1

517. $y = 1$

518. (a) $\frac{5}{4}(x - 4) = y - 2$ and $\frac{4}{5}(x - 2) = y - 4$ (b) 0 and $3\sqrt[3]{2}$ (c) 0 and $3\sqrt[3]{4}$

519. $(\pm\sqrt{7}, 0)$; slope is -2

520. $(3, -1)$

521. (a) $\dfrac{3x^y - y^2}{2xy - x^3}$ (b) at $(1, -2)$ the tangent is $2(x - 1) = y + 2$, at $(1, 3)$ the tangent is $y = 3$ (c) $\sqrt[5]{-24}$

530. $AC = \frac{5}{4}\sqrt{29}$, $BC = \frac{25}{2}$

531. $16\sqrt{3}$

532. 50π

533. 15

534. $3\sqrt[3]{9}$

535. $6\sqrt{3}$

536. $2\sqrt{3}$

545. (a) 34994 dollars/week (b) 200 dollars/week (c) 34794 dollars/week

546. $\frac{5}{16}\sqrt{3}$ m/hr

547. 18 m/sec

548. 3 ft/sec

550. $(\frac{1}{4}, \frac{1}{2})$

551. (a) $s = \frac{7}{8}d$ (b) $\frac{35}{8}$ ft/sec

552. 1 ft/min; 40π ft^2/min

553. ≈ 7.1 in/min

554. 12 in^3/sec

555. (a) $\pm\frac{5}{3}$ units/sec (b) ± 24

572. $\ln x$

579. $\dfrac{2^x \ln\frac{2}{5} + \ln 5}{5^x}$

582. $\dfrac{3}{5(3x - 2)}$

584. $\dfrac{x\ln x - x + 2}{x(\ln x)^2}$