

## Day 4 Homework

Use your calculator on problems 10 and 13c only.

- If  $x = t^2 - 1$  and  $y = e^{t^3}$ , find  $\frac{dy}{dx}$ .
- If a particle moves in the  $xy$ -plane so that at any time  $t > 0$ , its position vector is  $\langle \ln(t^2 + 5t), 3t^2 \rangle$ , find its velocity vector at time  $t = 2$ .
- A particle moves in the  $xy$ -plane so that at any time  $t$ , its coordinates are given by  $x = t^5 - 1$  and  $y = 3t^4 - 2t^3$ . Find its acceleration vector at  $t = 1$ .
- If a particle moves in the  $xy$ -plane so that at time  $t$  its position vector is  $\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \rangle$ , find the velocity vector at time  $t = \frac{\pi}{2}$ .
- A particle moves on the curve  $y = \ln x$  so that its  $x$ -component has derivative  $x'(t) = t + 1$  for  $t \geq 0$ . At time  $t = 0$ , the particle is at the point  $(1, 0)$ . Find the position of the particle at time  $t = 1$ .
- A particle moves in the  $xy$ -plane in such a way that its velocity vector is  $\langle 1 + t, t^3 \rangle$ . If the position vector at  $t = 0$  is  $\langle 5, 0 \rangle$ , find the position of the particle at  $t = 2$ .
- A particle moves along the curve  $xy = 10$ . If  $x = 2$  and  $\frac{dy}{dt} = 3$ , what is the value of  $\frac{dx}{dt}$ ?
- The position of a particle moving in the  $xy$ -plane is given by the parametric equations  $x = t^3 - \frac{3}{2}t^2 - 18t + 5$  and  $y = t^3 - 6t^2 + 9t + 4$ . For what value(s) of  $t$  is the particle at rest?
- A curve  $C$  is defined by the parametric equations  $x = t^3$  and  $y = t^2 - 5t + 2$ . Write the equation of the line tangent to the graph of  $C$  at the point  $(8, -4)$ .
- A particle moves in the  $xy$ -plane so that the position of the particle is given by  $x(t) = 5t + 3\sin t$  and  $y(t) = (8 - t)(1 - \cos t)$ . Find the velocity vector at the time when the particle's horizontal position is  $x = 25$ .
- The position of a particle at any time  $t \geq 0$  is given by  $x(t) = t^2 - 3$  and  $y(t) = \frac{2}{3}t^3$ .
  - Find the magnitude of the velocity vector at time  $t = 5$ .

- (b) Find the total distance traveled by the particle from  $t = 0$  to  $t = 5$ .
- (c) Find  $\frac{dy}{dx}$  as a function of  $x$ .
12. Point  $P(x, y)$  moves in the  $xy$ -plane in such a way that  $\frac{dx}{dt} = \frac{1}{t+1}$  and  $\frac{dy}{dt} = 2t$  for  $t \geq 0$ .
- (a) Find the coordinates of  $P$  in terms of  $t$  given that, when  $t = 1$ ,  $x = \ln 2$  and  $y = 0$ .
- (b) Write an equation expressing  $y$  in terms of  $x$ .
- (c) Find the average rate of change of  $y$  with respect to  $x$  as  $t$  varies from 0 to 4.
- (d) Find the instantaneous rate of change of  $y$  with respect to  $x$  when  $t = 1$ .
13. Consider the curve  $C$  given by the parametric equations  $x = 2 - 3\cos t$  and  $y = 3 + 2\sin t$ , for  $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ .
- (a) Find  $\frac{dy}{dx}$  as a function of  $t$ .
- (b) Find the equation of the tangent line at the point where  $t = \frac{\pi}{4}$ .
- (c) The curve  $C$  intersects the  $y$ -axis twice. Approximate the length of the curve between the two  $y$ -intercepts.

### Answers to Day 4 Homework

1. 
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}[e^{t^3}]}{\frac{d}{dt}[t^2 - 1]} = \frac{3t^2 e^{t^3}}{2t} = \frac{3te^{t^3}}{2}.$$
2. 
$$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle \frac{2t+5}{t^2+5t}, 6t \right\rangle \text{ so } v(2) = \left\langle \frac{9}{14}, 12 \right\rangle.$$
3. 
$$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle 5t^4, 12t^3 - 6t^2 \rangle.$$
- $$a(t) = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \langle 20t^3, 36t^2 - 12t \rangle, \text{ so } a(1) = \langle 20, 24 \rangle.$$
4. 
$$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 3\cos\left(3t - \frac{\pi}{2}\right), 6t \right\rangle \text{ so } v\left(\frac{\pi}{2}\right) = \langle 3\cos\pi, 3\pi \rangle = \langle -3, 3\pi \rangle.$$

5.  $x(t) = \int (t+1) dt = \frac{t^2}{2} + t + C.$

$x(0) = 1 = C$  so  $x(t) = \frac{t^2}{2} + t + 1.$

Since  $x(1) = \frac{5}{2}$  and  $y(1) = \ln\left(\frac{5}{2}\right)$ , Position =  $\left(\frac{5}{2}, \ln\left(\frac{5}{2}\right)\right).$

Or, since  $x(1) = 1 + \int_0^1 (t+1) dt = \frac{5}{2}$  and  $y(1) = \ln\left(\frac{5}{2}\right)$ , Position =  $\left(\frac{5}{2}, \ln\left(\frac{5}{2}\right)\right).$

6.  $x(t) = \int (1+t) dt = t + \frac{t^2}{2} + C.$   $x(0) = 5$  so  $C = 5$  and  $x(t) = t + \frac{t^2}{2} + 5.$

$y(t) = \int t^3 dt = \frac{t^4}{4} + D.$   $y(0) = 0$  so  $D = 0$  and  $y(t) = \frac{t^4}{4}.$

Position vector =  $\left\langle t + \frac{t^2}{2} + 5, \frac{t^4}{4} \right\rangle.$  At  $t = 2$ , Position =  $(9, 4).$

Or, since  $x(2) = 5 + \int_0^2 (1+t) dt = 9$  and  $y(2) = 0 + \int_0^2 t^3 dt = 4,$

Position =  $(9, 4).$

7. When  $x = 2, y = 5.$

$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0$$

$$(2)(3) + 5 \frac{dx}{dt} = 0 \text{ so } \frac{dx}{dt} = -\frac{6}{5}$$

Or find that  $\frac{dy}{dx} = -\frac{10}{x^2}.$  Then substituting into  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  gives  $\frac{-10}{4} = \frac{3}{\frac{dx}{dt}}$  so that

$$\frac{dx}{dt} = -\frac{6}{5}.$$

8.  $x'(t) = 3t^2 - 3t - 18 = 3(t-3)(t+2) = 0$  when  $t = 3$  and  $t = -2.$

$y'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$  when  $t = 3$  and  $t = 1.$

The particle is at rest when  $v(t) = \langle 0, 0 \rangle$  so at rest when  $t = 3.$

$$\begin{aligned}
 9. \quad t^3 &= 8 & t^2 - 5t + 2 &= -4 \\
 t = 2 & & t^2 - 5t + 6 &= 0 \\
 & & (t-3)(t-2) &= 0 \\
 & & t &= 3, t = 2
 \end{aligned}$$

At  $(8, -4)$  when  $t = 2$

$$\left. \frac{dy}{dx} \right|_{t=2} = \left. \frac{2t-5}{3t^2} \right|_{t=2} = -\frac{1}{12}$$

Tangent line equation:  $y + 4 = -\frac{1}{12}(x - 8)$

10.  $5t + 3\sin t = 25$  when  $t = 5.445755\dots$

$$v(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle 5 + 3\cos t, -1 + \cos t + (8-t)\sin t \rangle$$

$$v(5.445755\dots) = \langle 7.008, -2.228 \rangle$$

11. (a) Magnitude when  $t = 5$  is  $\left. \sqrt{(2t)^2 + (2t^2)^2} \right|_{t=5} = \sqrt{2600}$  or  $10\sqrt{26}$

(b)

$$\text{Distance} = \int_0^5 \sqrt{(2t)^2 + (2t^2)^2} dt = \int_0^5 2t\sqrt{1+t^2} dt = \frac{2}{3} (1+t^2)^{\frac{3}{2}} \Big|_0^5 = \frac{2}{3} \left( 26^{\frac{3}{2}} - 1 \right)$$

(c)  $\frac{dy}{dx} = \frac{2t^2}{2t} = t = \sqrt{x+3}$

12. (a)  $x(t) = \int \frac{1}{t+1} dt = \ln(t+1) + C$ . When  $t=1, x = \ln 2$  so  $C = 0$ .

$$x(t) = \ln(t+1)$$

$y(t) = \int 2t dt = t^2 + D$ . When  $t=1, y=0$  so  $D = -1$ .

$$y(t) = t^2 - 1$$

$$(x, y) = (\ln(t+1), t^2 - 1)$$

(b)  $t+1 = e^x$  so  $t = e^x - 1$  and  $y = (e^x - 1)^2 - 1 = e^{2x} - 2e^x$ .

(c) Average rate of change  $= \frac{y(b) - y(a)}{x(b) - x(a)} = \frac{y(4) - y(0)}{x(4) - x(0)} = \frac{15 - (-1)}{\ln 5 - \ln 1} = \frac{16}{\ln 5}$

(d) Instantaneous rate of change  $= \left. \frac{dy}{dx} \right|_{t=1} = \left. \frac{2t}{\frac{1}{t+1}} \right|_{t=1} = 4$

13. (a)  $\frac{dy}{dx} = \frac{2 \cos t}{3 \sin t} = \frac{2}{3} \cot t$

(b)  $y - (3 + \sqrt{2}) = \frac{2}{3} \left( x - \left( 2 - \frac{3\sqrt{2}}{2} \right) \right)$

(c)  $x = 0$  when  $t = -0.84106867\dots, 0.84106867\dots$

$$\text{length} = \int_{-0.841\dots}^{0.841\dots} \sqrt{(3 \sin t)^2 + (2 \cos t)^2} dt = 3.756 \text{ or } 3.757$$