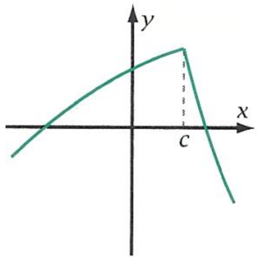
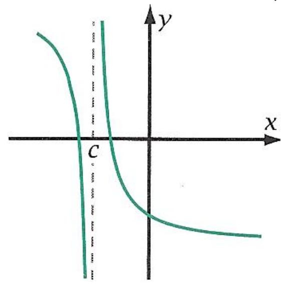
**AP Calculus AB** | **Name:**
**Section 4.6: Differentiability & Continuity** | **Period:** | **Date:**
**Practice Exercises** | **Score:** | **/ 5 Points**

**For exercises 1-6, state whether the function is continuous, differentiable, both, or neither at $x = c$.**

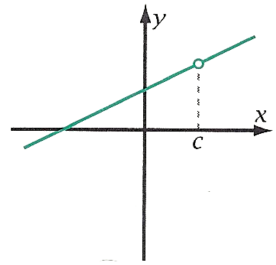
**1.**



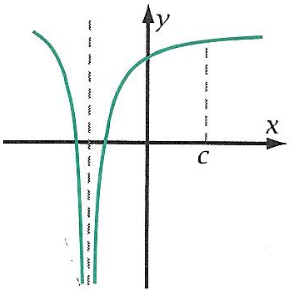

**2.**



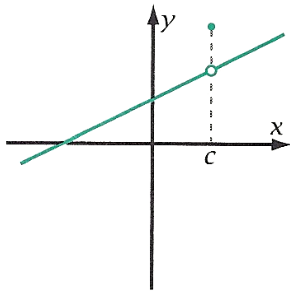

**3.**



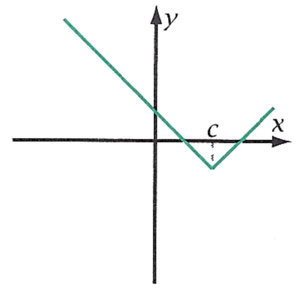

**4.**




**5.**



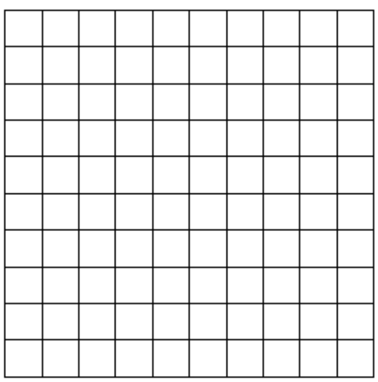

**6.**



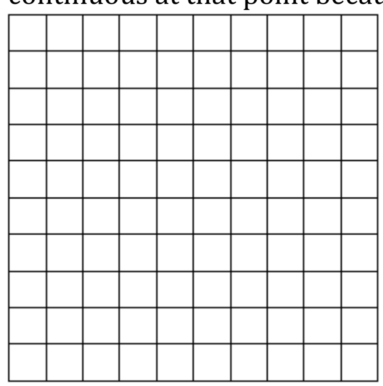

**For exercises 7-10,**

- **a. Sketch the graph of a function that has the indicated features.**
- **b. Write the equation for a function that has these features.**

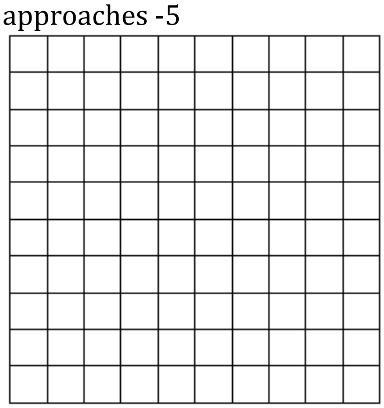
**7.** Is differentiable and continuous at the point (3, 5)



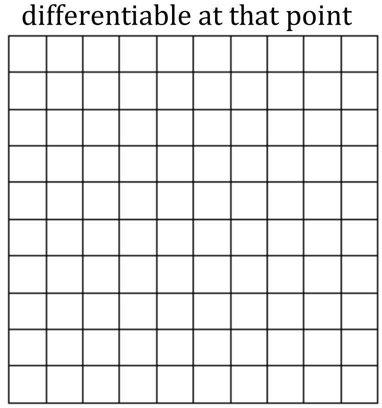
**8.** Has a finite limit as $x$ approaches 6, but is not continuous at that point because $f(6)$ is undefined



**9.** Has a value for $f(-5)$ but has no limit as $x$ approaches -5



**10.** Is continuous at the point (4, 7) but is not differentiable at that point



**For exercises 11-12, sketch the graph. State whether the function is differentiable, continuous, neither, or both at the indicated value of $x = c$.**

**11.** $f(x) = |x - 3|, \quad c = 3$

**12.** $f(x) = \sin x, \quad c = 1$

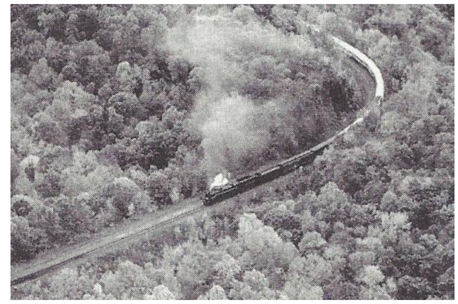
**For exercises 13-15, use one-sided limits to find the values of the constants *a* and *b* that make the piecewise function differentiable at the point where the rule for the function changes. Check your answer by plotting the graph. Sketch the results.**

**13.** $f(x) = \begin{cases} x^3, & \text{if } x < 1 \\ a(x-2)^2 + b, & \text{if } x \geq 1 \end{cases}$

**14.** $f(x) = \begin{cases} ax^2 + 10, & \text{if } x < 2 \\ x^2 - 6x + b, & \text{if } x \geq 2 \end{cases}$

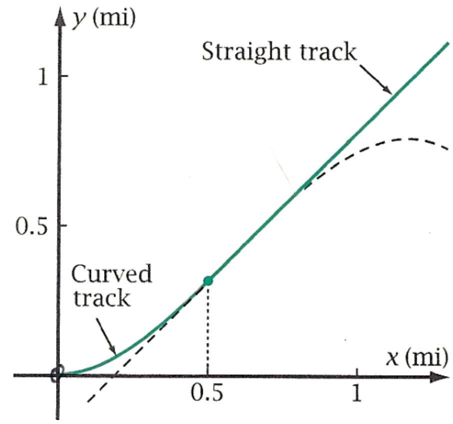
**15.** $f(x) = \begin{cases} e^{ax}, & \text{if } x \leq 1 \\ b + \ln x, & \text{if } x > 1 \end{cases}$

**16.** *Railroad Curve Problem:* Curves on a railroad track are in the shape of cubic parabolas. Such "parabola tracks" have the property that the curvature starts as zero at a particular point and increases gradually, easing the locomotive into the curve slowly so that it is less likely to derail. The figure (below right) shows curved and straight sections of track defined by the piecewise function



$$y = \begin{cases} ax^3 + bx^2 + cx + d, & \text{if } 0 \le x \le 0.5 \\ x + k, & \text{if } x > 0.5 \end{cases}$$

where $x$ and $y$ are coordinates in miles. The dashed portions show where the cubic and linear functions would go if they extended into other parts of the domain.




**a.** The curved left branch of the graph contains the origin and has $y' = 0$ at that point. At the transition point where $x = 0.5$, $y' = 1$ so that the curve goes the same direction as the straight section. At this point, $y'' = 0$ so that the curvature is zero. Find the coefficients *a, b, c,* and *d* in the cubric branch of the function.

b. Find the value of $k$ in the linear branch of the function that makes the piecewise function continuous.