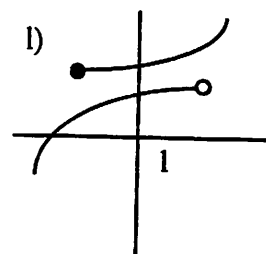
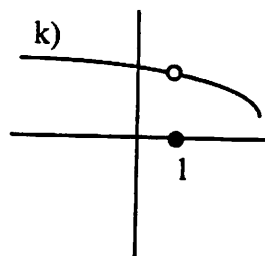
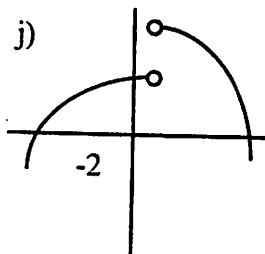
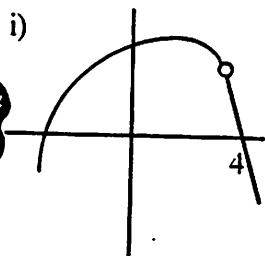
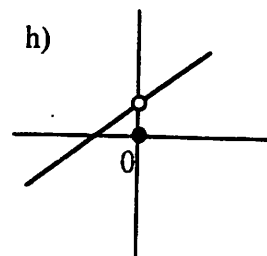
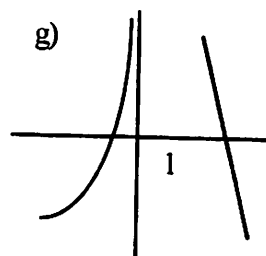
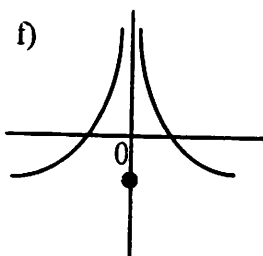
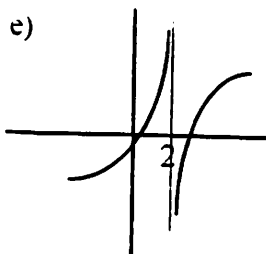
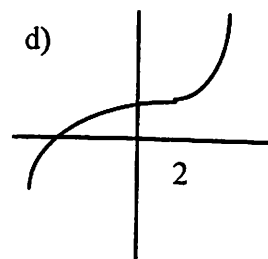
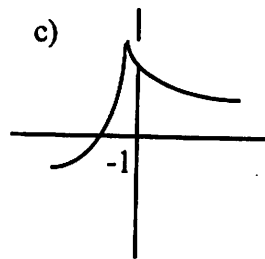
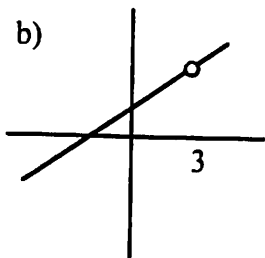
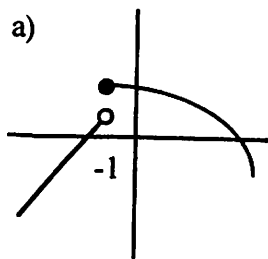


## Continuity and Differentiability - Homework

In the following graphs determine if the function  $f(x)$  is continuous at the marked value of  $c$ , and if not, determine for which of the 3 rules of continuity the function fails.



2. Find the value of  $x$  where the function is discontinuous.

a.  $f(x) = x^3 + 3^x$

b.  $f(x) = \frac{5}{x^2 - 81}$

c.  $f(x) = \frac{x^2 + 2x - 24}{x^2 - 36}$

d.  $f(x) = \tan x$

3. Find whether the function is continuous at the value where the rule for the function changes.

a.  $f(x) = \begin{cases} 8 - x^2, & x < 2 \\ 6 - x, & x \geq 2 \end{cases}$

b.  $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 1 + x, & x \geq 1 \end{cases}$

c.  $f(x) = \begin{cases} 2^x, & x < 3 \\ 10 - x, & x \geq 3 \end{cases}$

d.  $f(x) = \begin{cases} 2^{-x}, & x < -1 \\ x+3, & x \geq -1 \end{cases}$

e.  $f(x) = \begin{cases} \frac{1}{x-2}, & x < 2 \\ 3, & x = 2 \\ x+1, & x > 2 \end{cases}$

f.  $f(x) = \begin{cases} \frac{x^3 - x}{x^2 - x}, & x \neq 0, x \neq 1 \\ 3, & x = 0 \\ 2, & x = 1 \end{cases}$

4. Find the value of the constant  $a$  that makes the function continuous.

a.  $f(x) = \begin{cases} 0.4x+2, & x > 1 \\ 0.3x+a, & x \leq 1 \end{cases}$

b.  $f(x) = \begin{cases} x^2, & x > 2 \\ a-x, & x \leq 2 \end{cases}$

c.  $f(x) = \begin{cases} 9-x^2, & x > 2 \\ ax, & x \leq 2 \end{cases}$

d.  $f(x) = \begin{cases} ax+5, & x < -1 \\ ax^2, & x \geq -1 \end{cases}$

e.  $f(x) = \begin{cases} 0.4x+a^2, & x < -1 \\ ax+1.6, & x \geq -1 \end{cases}$

f.  $f(x) = \begin{cases} a^2 - x^2, & x < 2 \\ 1.5ax, & x \geq 2 \end{cases}$

Let  $a$  and  $b$  stand for constants and let  $f(x) = \begin{cases} b-x, & x < 1 \\ a(x-2)^2, & x \geq 1 \end{cases}$

a. Find an equation relating  $a$  and  $b$  if  $f$  is to be continuous at  $x = 1$ .

b. Find  $b$  if  $a = -1$ . Graph and show that the function is continuous

c. Find another value of  $a$ ,  $b$  where  $f$  is continuous.

6. Graph the function  $f(x) = x + 4 + \frac{10^{-25}}{x-2}$  and find what appears to be the limit of  $f(x)$  as  $x$  approaches 2.

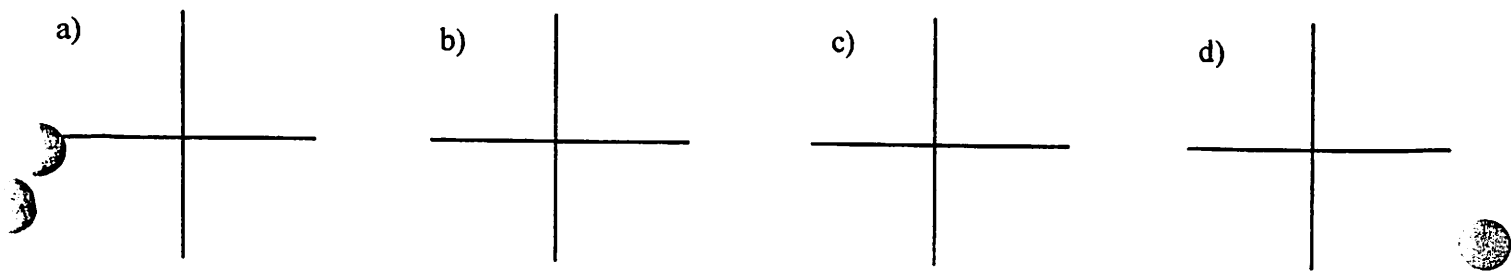
7. Sketch a function having the following attributes.

a) has a value of  $f(2)$ , a limit as  $x$  approaches 2, but is not continuous at  $x = 2$ .

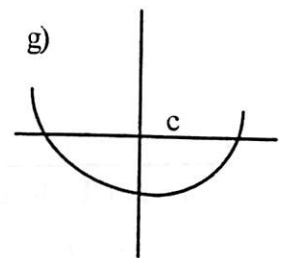
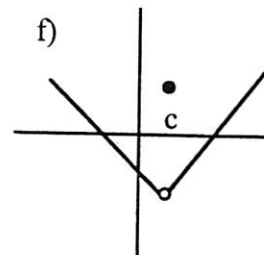
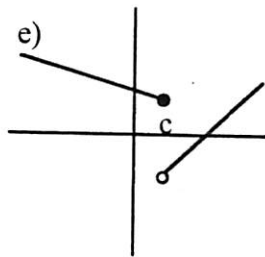
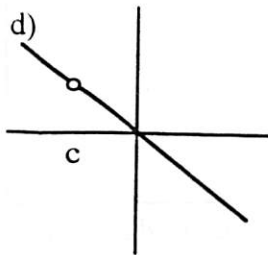
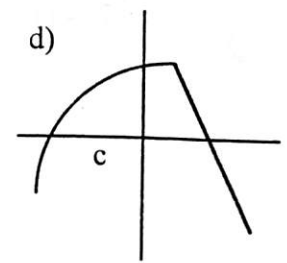
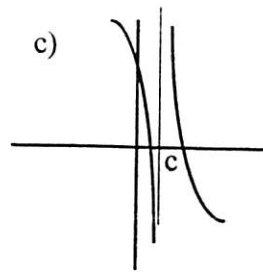
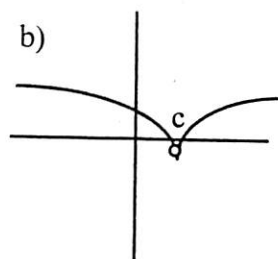
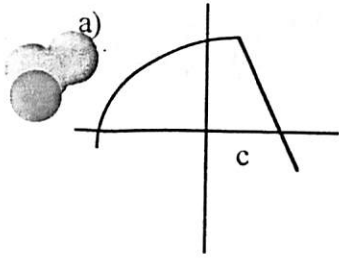
b) has a step discontinuity at  $x = 3$  where  $f(3) = 7$

c.  $\lim_{x \rightarrow 4} f(x) = -2$  but the function is not continuous at  $x = 4$ .

d. the value of  $f(-2) = 3$  but there is no limit of  $f(x)$  as  $x$  approaches -2 and no vertical asymptote there.



8. For the following, state whether the function is continuous, differentiable, both, or neither at  $x = c$ .



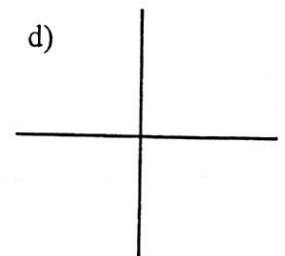
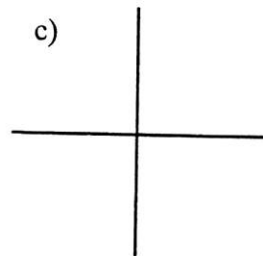
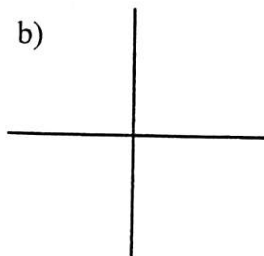
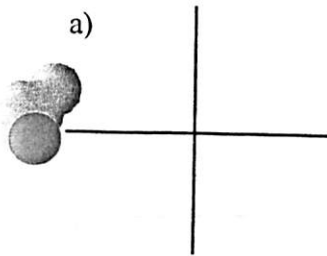
9. Sketch a function having the following attributes.

a. is differentiable and continuous at point  $(2, 4)$

c. has a cusp at the point  $(-1, 3)$

b. is continuous at  $(-3, 1)$  but not differentiable there

d. is differentiable at  $(2, -4)$  but not continuous there.

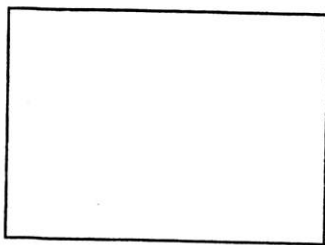


10. For each function,  $f(x)$ , show work to determine whether the function is continuous or non-continuous, differentiable, or non-differentiable, and sketch the curve. Show work necessary to prove your statements.

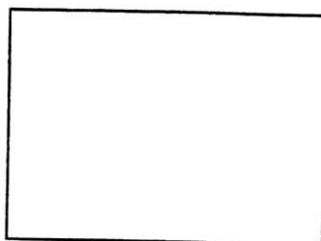
a.  $f(x) = \begin{cases} x^2, & x \geq 0 \\ x, & x < 0 \end{cases}$

b.  $f(x) = \begin{cases} x^2 + 1, & x \geq 0 \\ x^3 + 1, & x < 0 \end{cases}$

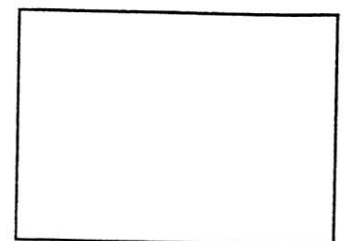
c.  $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 2x + 2, & x \geq 1 \end{cases}$



cont, diff, both, neither



cont, diff, both, neither



cont, diff, both, neither

$$d. f(x) = \begin{cases} x^2 + x - 7, & x \geq 2 \\ 5x - 11, & x < 2 \end{cases}$$

cont, diff, both, neither

$$g. f(x) = \begin{cases} \sin(x), & x > 0 \\ x, & x \leq 0 \end{cases}$$

cont, diff, both, neither

$$e. f(x) = \begin{cases} x^4 - 2x^2, & x > 1 \\ -1, & x \leq 1 \end{cases}$$

cont, diff, both, neither

$$h. f(x) = \begin{cases} \cos(x), & x \geq 0 \\ 1 - x^2, & x < 0 \end{cases}$$

cont, diff, both, neither

$$f. f(x) = \begin{cases} \sqrt{x} - 3, & x > 1 \\ \frac{1}{2}x - \frac{5}{2}, & x \leq 1 \end{cases}$$

cont, diff, both, neither

$$i. f(x) = \begin{cases} 3 + (x+2)^{1/3}, & x \geq -2 \\ 3 - (x+2)^{2/3}, & x < -2 \end{cases}$$

cont, diff, both, neither

11. Find the values of  $a$  and  $b$  that make the function  $f(x)$  differentiable.

$$a. f(x) = \begin{cases} x^2, & x \geq 1 \\ a(x-2)^2 + b, & x < 1 \end{cases}$$

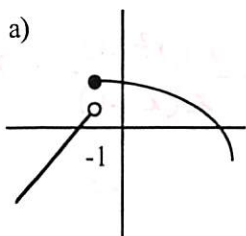
$$b. f(x) = \begin{cases} ax^2 + 10, & x \geq 2 \\ x^2 - 6x + b, & x < 2 \end{cases}$$

$$c. f(x) = \begin{cases} a/x, & x \geq 1 \\ 12 - bx^2, & x < 1 \end{cases}$$

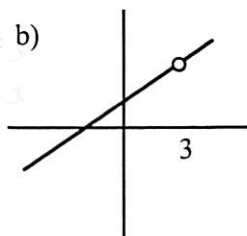
# ANSWERS

## Continuity and Differentiability - Homework

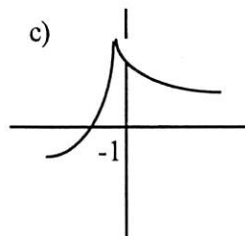
1. In the following graphs determine if the function  $f(x)$  is continuous at the marked value of  $c$ , and if not, determine for which of the 3 rules of continuity the function fails.



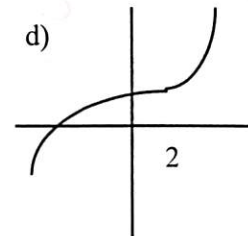
nc step 1



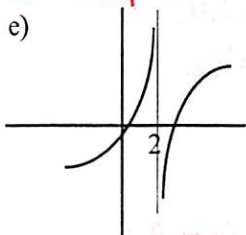
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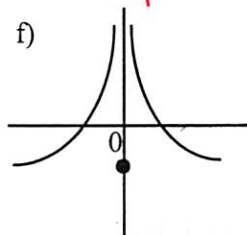
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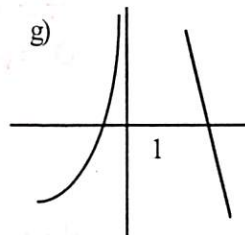
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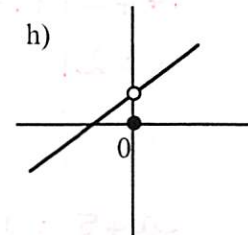
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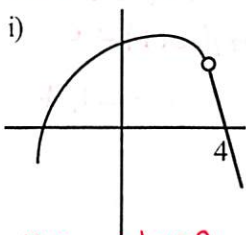
nc step 3



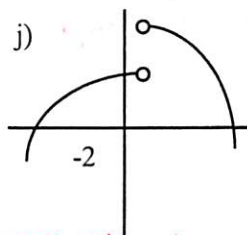
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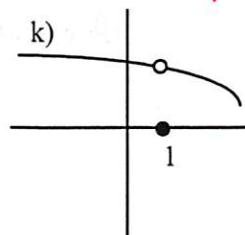
nc step 3



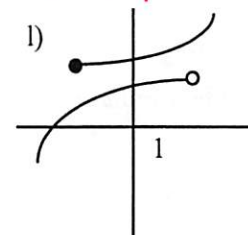
nc step 2



nc step 1



nc step 3



nc nd

2. Find the value of  $x$  where the function is discontinuous.

a.  $f(x) = x^3 + 3^x$

c

b.  $f(x) = \frac{5}{x^2 - 81}$

$x = \pm 9$

c.  $f(x) = \frac{x^2 + 2x - 24}{x^2 - 36}$

$x = \pm 6$

d.  $f(x) = \tan x$

$x = \frac{\pi}{2} + n\pi \quad n \in \mathbb{Z}$

3. Find whether the function is continuous at the value where the rule for the function changes.

a.  $f(x) = \begin{cases} 8 - x^2, & x < 2 \\ 6 - x, & x \geq 2 \end{cases}$

c

b.  $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 1 + x, & x \geq 1 \end{cases}$

nc

c.  $f(x) = \begin{cases} 2^x, & x < 3 \\ 10 - x, & x \geq 3 \end{cases}$

nc

$$d. f(x) = \begin{cases} 2^{-x}, & x < -1 \\ x+3, & x \geq -1 \end{cases}$$

C

$$e. f(x) = \begin{cases} \frac{1}{x-2}, & x < 2 \\ 3, & x = 2 \\ x+1, & x > 2 \end{cases}$$

nc

$$f. f(x) = \begin{cases} \frac{x^3-x}{x^2-x}, & x \neq 0, x \neq 1 \\ 3, & x = 0 \\ 2, & x = 1 \end{cases}$$

nc

$$\frac{x^3-x}{x^2-x} = \frac{x(x^2-1)}{x(x-1)} = x+1$$

4. Find the value of the constant  $a$  that makes the function continuous.

$$a. f(x) = \begin{cases} 0.4x+2, & x > 1 \\ 0.3x+a, & x \leq 1 \end{cases}$$

$$0.4+2 = 0.3+a$$

$$a = 2.1$$

$$b. f(x) = \begin{cases} x^2, & x > 2 \\ a-x, & x \leq 2 \end{cases}$$

$$4 = a - 2$$

$$a = 6$$

$$c. f(x) = \begin{cases} 9-x^2, & x > 2 \\ ax, & x \leq 2 \end{cases}$$

$$5 = 2a$$

$$a = 5/2$$

$$d. f(x) = \begin{cases} ax+5, & x < -1 \\ ax^2, & x \geq -1 \end{cases}$$

$$-a+5 = a$$

$$a = 5/2$$

$$e. f(x) = \begin{cases} 0.4x+a^2, & x < -1 \\ ax+1.6, & x \geq -1 \end{cases}$$

$$-0.4+a^2 = -a+1.6$$

$$a^2+a-2 = 0$$

$$(a+2)(a-1) = 0 \quad a = -2, 1$$

$$f. f(x) = \begin{cases} a^2-x^2, & x < 2 \\ 1.5ax, & x \geq 2 \end{cases}$$

$$a^2-4 = 3a$$

$$a^2-3a-4 = 0$$

$$(a-4)(a+1) = 0$$

$$a = -1, 4$$

5. Let  $a$  and  $b$  stand for constants and let

$$f(x) = \begin{cases} b-x, & x < 1 \\ a(x-2)^2, & x \geq 1 \end{cases}$$

a. Find an equation relating  $a$  and  $b$  if  $f$  is to be continuous at  $x = 1$ .

$$b-1 = a$$

b. Find  $b$  if  $a = -1$ . Graph and show that the function is continuous

$$b = 0$$

c. Find another value of  $a$ ,  $b$  where  $f$  is continuous.

6. Graph the function  $f(x) = x+4 + \frac{10^{-25}}{x-2}$  and find what appears to be the limit of  $f(x)$  as  $x$  approaches 2.

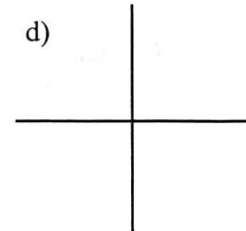
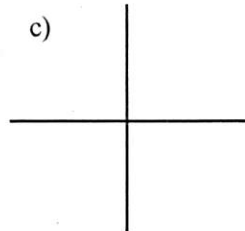
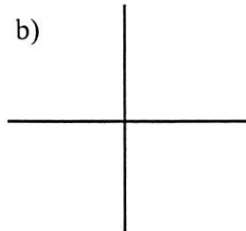
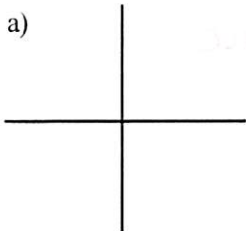
7. Sketch a function having the following attributes.

a) has a value of  $f(2)$ , a limit as  $x$  approaches 2, but is not continuous at  $x = 2$ .

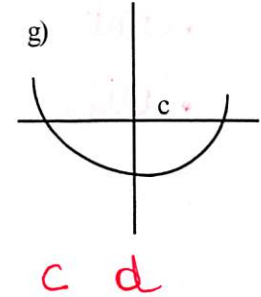
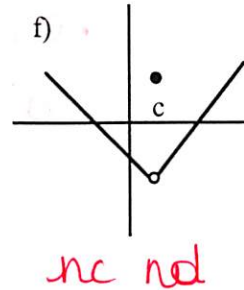
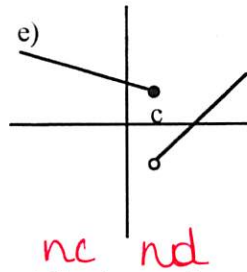
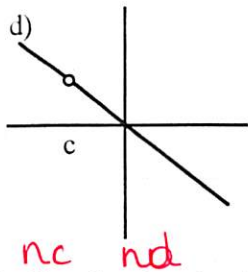
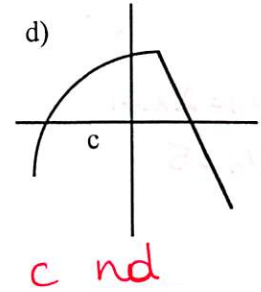
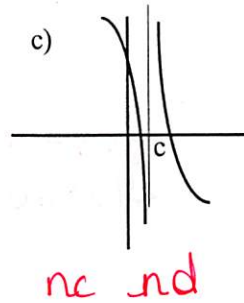
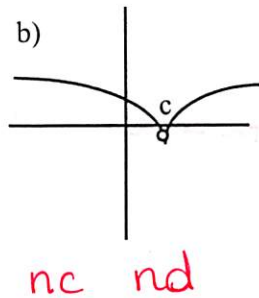
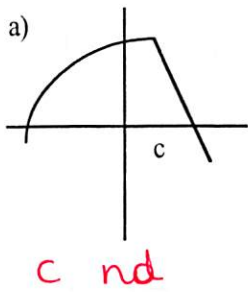
c.  $\lim_{x \rightarrow 4} f(x) = -2$  but the function is not continuous at  $x = 4$ .

b. has a step discontinuity at  $x = 3$  where  $f(3) = 7$

d. the value of  $f(-2) = 3$  but there is no limit of  $f(x)$  as  $x$  approaches  $-2$  and no vertical asymptote there.



8. For the following, state whether the function is continuous, differentiable, both, or neither at  $x = c$ .



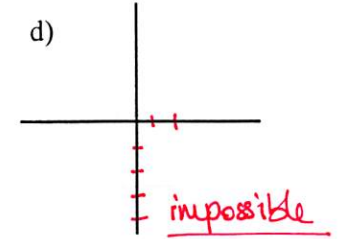
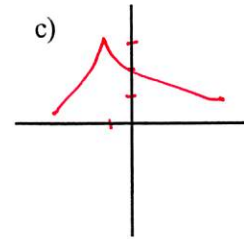
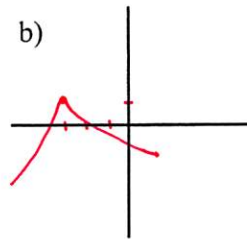
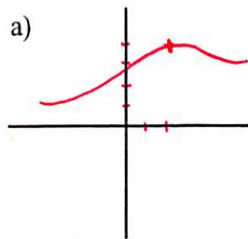
9. Sketch a function having the following attributes.

a. is differentiable and continuous at point  $(2, 4)$

c. has a cusp at the point  $(-1, 3)$

b. is continuous at  $(-3, 1)$  but not differentiable there

d. is differentiable at  $(2, -4)$  but not continuous there.



10. For each function,  $f(x)$ , show work to determine whether the function is continuous or non-continuous, differentiable, or non-differentiable, and sketch the curve. Show work necessary to prove your statements.

a.  $f(x) = \begin{cases} x^2, & x \geq 0 \\ x, & x < 0 \end{cases}$

$f'(x) = 2x \xrightarrow{x \rightarrow 0} 0$   
 $f'(x) = 1$

• cont  
 • nd

cont, diff, both, neither

b.  $f(x) = \begin{cases} x^2 + 1, & x \geq 0 \\ x^3 + 1, & x < 0 \end{cases}$

$f'(x) = 2x \xrightarrow{x \rightarrow 0} 0$   
 $f'(x) = 3x^2 \xrightarrow{x \rightarrow 0} 0$

• cont  
 • diff

cont, diff, both, neither

c.  $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 2x + 2, & x \geq 1 \end{cases}$

• nc  
 ↓  
 • nd

cont, diff, both, neither



$$d. f(x) = \begin{cases} x^2 + x - 7, & x \geq 2 \\ 5x - 11, & x < 2 \end{cases}$$

$$f'(x) = 2x + 1$$

$$f'(x) = 5$$

• cont  
• diff

cont, diff, both, neither

$$e. f(x) = \begin{cases} x^4 - 2x^2, & x > 1 \\ -1, & x \leq 1 \end{cases}$$

$$f'(x) = 4x^3 - 4x \rightarrow 0$$

$$f'(x) = 0$$

• cont  
• diff

cont, diff, both, neither

$$f. f(x) = \begin{cases} \sqrt{x} - 3, & x > 1 \\ \frac{1}{2}x - \frac{5}{2}, & x \leq 1 \end{cases}$$

cont, diff, both, neither

cont, diff, both, neither

$$g. f(x) = \begin{cases} \sin(x), & x > 0 \\ x, & x \leq 0 \end{cases}$$

$$h. f(x) = \begin{cases} \cos(x), & x \geq 0 \\ 1 - x^2, & x < 0 \end{cases}$$

$$i. f(x) = \begin{cases} 3 + (x+2)^{1/3}, & x \geq -2 \\ 3 - (x+2)^{2/3}, & x < -2 \end{cases}$$

cont, diff, both, neither

cont, diff, both, neither

cont, diff, both, neither

cont, diff, both, neither

cont, diff, both, neither

cont, diff, both, neither

\* 11. Find the values of  $a$  and  $b$  that make the function  $f(x)$  differentiable.

$$a. f(x) = \begin{cases} x^3, & x \geq 1 \\ a(x-2)^2 + b, & x < 1 \end{cases}$$

• cont:  $a = 1$

• diff:  $3x^2$   
 $2a(x-2)$  imp.

$$3 = -2a$$

$$a = -3/2$$

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$$b. f(x) = \begin{cases} ax^2 + 10, & x \geq 2 \\ x^2 - 6x + b, & x < 2 \end{cases}$$

• cont:  $4a + 10 = -8 + b$   
 $b = 4a + 18$

• diff:  $2ax$   
 $2x - 6$

$$4a = -2$$

$$a = -\frac{1}{2}$$

$$\Rightarrow b = 16$$

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$$c. f(x) = \begin{cases} a/x, & x \geq 1 \\ 12 - bx^2, & x < 1 \end{cases}$$

• cont:  $a = 12 - b$

• diff:  $-a = 12 - 2b$

$$+12 - b = 12 - 2b$$

$$b = 0$$

$$a = 12$$

Stu Schwartz

$$b = 4$$

$$a = 8$$