
Practice problems set 3:
Geometric and telescoping series

1. Determine whether the following geometric series converge or diverge. If they converge, find their values.

a) $\sum_{k=1}^{+\infty} 6(0.9)^{k-1}$

c) $\sum_{k=1}^{+\infty} \frac{12}{(-5)^{k+1}}$

e) $\sum_{k=1}^{+\infty} \frac{1}{4^{k/2}}$

b) $\sum_{n=1}^{+\infty} (-3)^{n-1}$

d) $\sum_{n=1}^{+\infty} \frac{(-3)^{n-1}}{4^n}$

2. For which values of x does the geometric series $\sum_{n=1}^{+\infty} (\ln x)^{n-1}$ converge?

3. Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \dots = 9$.

4. Determine whether the following telescoping series converge or diverge. If they converge, find their values.

a) $\sum_{n=2}^{+\infty} \frac{2}{n^2 - 1}$

c) $\sum_{n=2}^{+\infty} \left[\cos\left(\frac{1}{2n}\right) - \cos\left(\frac{1}{2n+2}\right) \right]$

b) $\sum_{n=1}^{+\infty} \frac{3}{n(n+3)}$

d) $\sum_{n=1}^{+\infty} \ln\left(\frac{n}{n+1}\right)$

Answers

1. a) Converges to 60. c) Converges to $\frac{2}{5}$ e) Converges to 1
b) Diverges (DNE) d) Converges to $\frac{1}{7}$
2. For x in the interval $(\frac{1}{e}, e)$
3. $b = \ln(8/9)$
4. a) Converges to $\frac{3}{2}$ c) Converges to $\cos(1/2) - 1$
b) Converges to $\frac{11}{6}$ d) Diverges to $-\infty$