CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

### A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If $y = x \sin x$, then $\dfrac{dy}{dx} =$

   (A) $\sin x + \cos x$

   (B) $\sin x + x \cos x$

   (C) $\sin x - x \cos x$

   (D) $x(\sin x + \cos x)$

   (E) $x(\sin x - \cos x)$

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

   (A) $(-\infty, -10]$ and $[10, \infty)$

   (B) $[-10, 10]$

   (C) $[0, 10]$ only

   (D) $[0, 10\sqrt{3}]$ only

   (E) $[0, \infty)$

GO ON TO THE NEXT PAGE.

3. $\int \sec x \tan x \, dx =$

   (A) $\sec x + C$

   (B) $\tan x + C$

   (C) $\dfrac{\sec^2 x}{2} + C$

   (D) $\dfrac{\tan^2 x}{2} + C$

   (E) $\dfrac{\sec^2 x \, \tan^2 x}{2} + C$

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

   (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim\limits_{x \to 2} f(x)$ exists.

(B) $\lim\limits_{x \to 3} f(x)$ exists.

(C) $\lim\limits_{x \to 4} f(x)$ exists.

(D) $\lim\limits_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3   (B) 6   (C) 9   (D) 18   (E) 27

GO ON TO THE NEXT PAGE.

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9   (B) 68.2   (C) 114.9   (D) 116.6   (E) 118.2

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} \dfrac{(2x + 1)(x - 2)}{x - 2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 5

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

   (A) $2e - 2$ (B) $2e$ (C) $\dfrac{e}{2} - 1$ (D) $\dfrac{e - 1}{2}$ (E) $e - 1$

GO ON TO THE NEXT PAGE.

11. Let f be the function defined by $f(x) = \sqrt{|x - 2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim\limits_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

12. Using the substitution $u = \sqrt{x}$, $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to which of the following?

(A) $2\displaystyle\int_1^{16} e^u\, du$

(B) $2\displaystyle\int_1^4 e^u\, du$

(C) $2\displaystyle\int_1^2 e^u\, du$

(D) $\dfrac{1}{2}\displaystyle\int_1^2 e^u\, du$

(E) $\displaystyle\int_1^4 e^u\, du$

GO ON TO THE NEXT PAGE.

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\, dx$?

(A) 2   (B) 6   (C) 8   (D) 10   (E) 12

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\dfrac{7}{\sqrt{5}}$ (B) $\dfrac{14}{\sqrt{5}}$ (C) $\dfrac{18}{\sqrt{5}}$ (D) $\dfrac{15}{\sqrt{21}}$ (E) $\dfrac{30}{\sqrt{21}}$

GO ON TO THE NEXT PAGE.

Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\, dt,$ which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

GO ON TO THE NEXT PAGE.

16. A particle moves along the *x*-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \dfrac{a + b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$

Graph of f

17. The figure above shows the graph of f. If $f(x) = \int_{2}^{x} g(t)\,dt,$ which of the following could be the graph

of $y = g(x)$?

(A)

(B)

(C)

(D)

(E)

GO ON TO THE NEXT PAGE.

18. $\displaystyle\lim_{h\to 0}\frac{\ln(4+h)-\ln(4)}{h}$ is

(A) 0   (B) $\dfrac{1}{4}$ (C) 1   (D) e (E) nonexistent

19. The function f is defined by $f(x)=\dfrac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2},\dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\dfrac{2}{3}\right)$

(E) There are no such points.

GO ON TO THE NEXT PAGE.

20. Let $f(x) = (2x + 1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\dfrac{2}{27}$ (B) $\dfrac{1}{54}$ (C) $\dfrac{1}{27}$ (D) $\dfrac{1}{6}$ (E) 6

---

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x - 5}$ (D) $y = \dfrac{5x}{1 - x}$ (E) $y = \dfrac{20x^2 - x}{1 + 4x^2}$

GO ON TO THE NEXT PAGE.

22. Let f be the function defined by $f(x) = \dfrac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\dfrac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

GO ON TO THE NEXT PAGE.

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \dfrac{2}{3}$?

    (A) -3 (B) $-\dfrac{3}{2}$ (C) $-\dfrac{1}{3}$ (D) 0 (E) There is no such k.

GO ON TO THE NEXT PAGE.

25. Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

   (A) $y = 2\cos x + 3$

   (B) $y = 2\cos x - 1$

   (C) $y = -2\cos x + 3$

   (D) $y = -2\cos x + 1$

   (E) $y = -2\cos x - 1$

GO ON TO THE NEXT PAGE.

26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\, dt$. Which of the following must be true on the interval $0 < x < 2$?

    (A) g is increasing, and the graph of g is concave up.

    (B) g is increasing, and the graph of g is concave down.

    (C) g is decreasing, and the graph of g is concave up.

    (D) g is decreasing, and the graph of g is concave down.

    (E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

GO ON TO THE NEXT PAGE.

27. If $(x + 2y) \cdot \dfrac{dy}{dx} = 2x - y,$ what is the value of $\dfrac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\dfrac{10}{3}$ (B) 0 (C) 2 (D) $\dfrac{10}{3}$ (E) Undefined

GO ON TO THE NEXT PAGE.

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

    (A) $-\sqrt{2}$　　　(B) -1　　　(C) 0　　　(D) 1　　　(E) $\sqrt{2}$

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1)   The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.
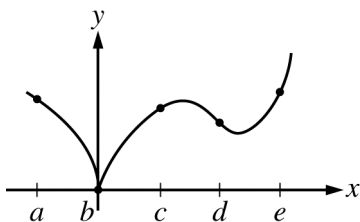
Graph of f

76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?

(A) a (B) b (C) c (D) d (E) e

GO ON TO THE NEXT PAGE.

77. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

---

78. The graph of $y = e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606   (B) 2   (C) 2.242   (D) 2.961   (E) 3.747

GO ON TO THE NEXT PAGE.

79. A particle moves along the *x*-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

(A) $\dfrac{a(8) - a(0)}{8}$

(B) $\dfrac{1}{8}\displaystyle\int_0^8 v(t)\, dt$

(C) $\dfrac{1}{8}\displaystyle\int_0^8 |v(t)|\, dt$

(D) $\dfrac{1}{2}\displaystyle\int_0^8 v(t)\, dt$

(E) $\dfrac{v(0) + v(8)}{2}$

GO ON TO THE NEXT PAGE.
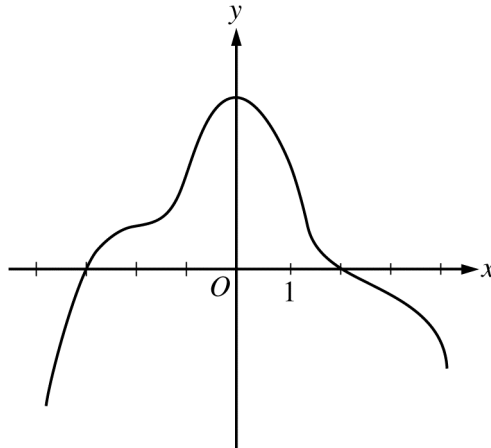
Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

    I. f has a relative minimum at $x = -3$.

    II. The graph of f has a point of inflection at $x = -2$.

    III. The graph of f is concave down for $0 < x < 4$.

(A) I only   (B) II only   (C) III only   (D) I and II only   (E) I and III only

GO ON TO THE NEXT PAGE.

81. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A)   380 gallons

(B)   420 gallons

(C)   829 gallons

(D) 1220 gallons

(E) 1376 gallons

82. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x$, then f has a local maximum at $x =$

(A) −2.314   (B) −1.332   (C) 0.350   (D) 0.829   (E) 1.234

GO ON TO THE NEXT PAGE.

$v(t)$

Velocity
(feet per second)

50
40
30
20
10

O 1 2 3 4 5 6 7 8 t
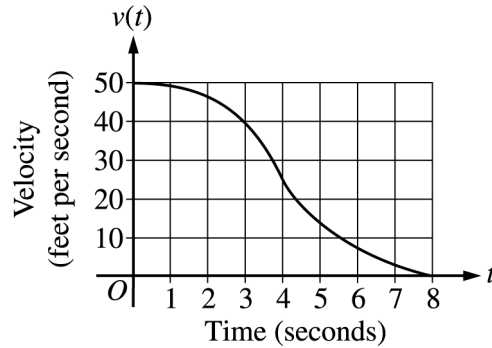
Time (seconds)

83. The graph above gives the velocity, v, in ft/sec, of a car for $0 \leq t \leq 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

(A) 21 ft   (B) 26 ft   (C) 180 ft   (D) 210 ft   (E) 260 ft

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

GO ON TO THE NEXT PAGE.
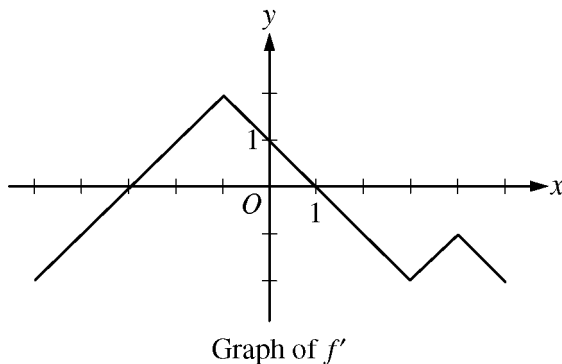
Graph of f'

85. The graph of f', the derivative of f, is shown in the figure above. The function f has a local maximum at $x =$

(A) −3   (B) −1   (C) 1   (D) 3   (E) 4

GO ON TO THE NEXT PAGE.

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)\,dt = 0$, which of the following could be a table of values for the function f ?

(A)

x	$f(x)$
4	-4
5	-3
7	0

(B)

x	$f(x)$
4	-4
5	-2
7	5

(C)

x	$f(x)$
4	-4
5	6
7	3

(D)

x	$f(x)$
4	0
5	0
7	0

(E)

x	$f(x)$
4	0
5	4
7	6

GO ON TO THE NEXT PAGE.
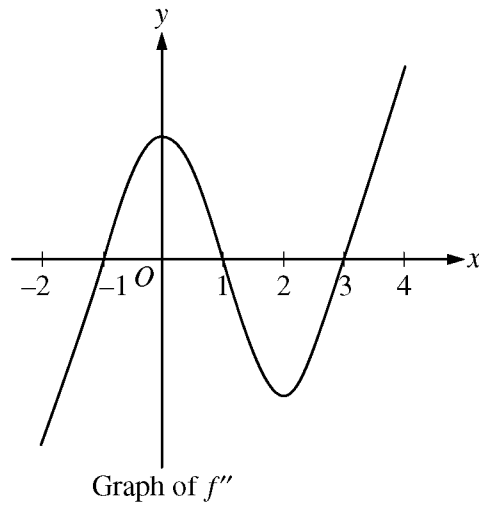
Graph of f''

87. The graph of f'', the second derivative of f, is shown above for $-2 \le x \le 4$. What are all intervals on which the graph of the function f is concave down?

(A) $-1 < x < 1$

(B) $\ 0 < x < 2$

(C) $\ 1 < x < 3$ only

(D) $-2 < x < -1$ only

(E) $-2 < x < -1$ and $1 < x < 3$

GO ON TO THE NEXT PAGE.
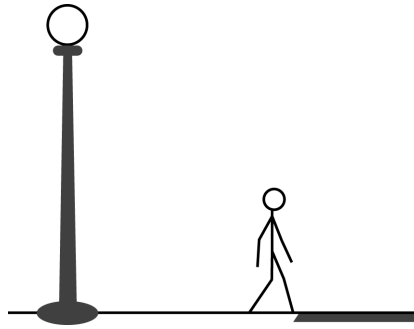
88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec   (B) 2.667 ft/sec   (C) 3.75 ft/sec   (D) 6 ft/sec   (E) 10 ft/sec

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713   (B) 1.134   (C) 6.134   (D) 6.710   (E) 11.710

GO ON TO THE NEXT PAGE.

90. Let f be a function such that $\int_6^{12} f(2x)\,dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t)\,dt = 5$

(B) $\int_{12}^{24} f(t)\,dt = 20$

(C) $\int_6^{12} f(t)\,dt = 5$

(D) $\int_6^{12} f(t)\,dt = 20$

(E) $\int_3^6 f(t)\,dt = 5$

x	−2	0	3	5	6
$f'(x)$	3	1	4	7	5

91. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of f is concave up.
(B) The graph of f has at least two points of inflection.
(C) f is increasing.
(D) f has no critical points.
(E) f has at least two relative extrema.

GO ON TO THE NEXT PAGE.

92. Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph of $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x-axis are squares. What is the volume of the solid?

(A) 0.129   (B) 0.300   (C) 0.333   (D) 0.700   (E) 1.271