1. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{3x-2} & \text{if } x \leqslant 1 \\ x^2+1 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

2. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{3}{x+2} - 1 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

3. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{-2x^2+3x+8} & \text{if } x \leqslant 1 \\ \frac{-9x}{x+2} + 6 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

4. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{-18x}{x+3} + 10 & \text{if } x \leqslant 3 \\ \frac{x}{x-2} - 1 & \text{if } x > 3 \end{cases}$$

Analize the continuity of this function at $x = 3$.

5. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{x^2-2x-15} & \text{if } x < -3 \\ 0 & \text{if } x = -3 \\ x^2-10 & \text{if } x > -3 \end{cases}$$

Analize the continuity of this function at $x = -3$.

6. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{-6x}{x} + 7 & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ -3x+8 & \text{if } x > 2 \end{cases}$$

Analize the continuity of this function at $x = 2$.

7. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} x^2 - x - 8 & \text{if } x < -2 \\ -2x^2 - x + 4 & \text{if } x \geqslant -2 \end{cases}$$

Analize the continuity of this function at $x = -2$.

8. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ \sqrt{x^2 + 3} & \text{if } x \geqslant 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

9. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} -2x^2 + 2x + 13 & \text{if } x \leqslant 3 \\ \sqrt{2x^2 - 3x - 9} & \text{if } x > 3 \end{cases}$$

Analize the continuity of this function at $x = 3$.

10. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{6}{x-3} + 3 & \text{if } x \leqslant 1 \\ -3x^2 - 2x + 4 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

Answers: 1. discontinuous 2. discontinuous 3. continuous 4. discontinuous 5. discontinuous 6. discontinuous 7. continuous 8. continuous 9. discontinuous 10. discontinuous

Solutions:

1.
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{3x-2} = \sqrt{3(1)-2} = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2+1 = (1)^2+1 = 2$

$f(1) = 1$

So $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) = f(1) \qquad \lim_{x\to 1^+} f(x) \neq f(1)$

Therefore, the function is discontinuous.

2.
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{3}{x+2} - 1 = \frac{3}{(1)+2} - 1 = 0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \sqrt{x} = \sqrt{(1)} = 1$

$f(1) = 1$

So $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) \neq f(1) \qquad \lim_{x\to 1^+} f(x) = f(1)$

Therefore, the function is discontinuous.

3.
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{-2x^2+3x+8} = \sqrt{-2(1)^2+3(1)+8} = 3$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{-9x}{x+2} + 6 = \frac{-9(1)}{(1)+2} + 6 = 3$

$f(1) = 3$

So $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) = f(1) \qquad \lim_{x\to 1^+} f(x) = f(1)$

Therefore, the function is continuous.

4.
$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{-18x}{x+3} + 10 = \frac{-18(3)}{(3)+3} + 10 = 1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x}{x-2} - 1 = \frac{(3)}{(3)-2} - 1 = 2$

$f(3) = 1$

So $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x) \qquad \lim_{x\to 3^-} f(x) = f(3) \qquad \lim_{x\to 3^+} f(x) \neq f(3)$

Therefore, the function is discontinuous.

5.
$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \sqrt{x^2-2x-15} = \sqrt{(-3)^2-2(-3)-15} = 0$

$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} x^2-10 = (-3)^2-10 = -1$

$f(-3) = 0$

So $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x) \qquad \lim_{x\to -3^-} f(x) = f(-3) \qquad \lim_{x\to -3^+} f(x) \neq f(-3)$

Therefore, the function is discontinuous.

6.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{-6x}{x} + 7 = \frac{-6(2)}{(2)} + 7 = 1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -3x + 8 = -3(2) + 8 = 2$

$f(2) = 2$

So $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ $\quad \lim_{x\to 2^-} f(x) \neq f(2)$ $\quad \lim_{x\to 2^+} f(x) = f(2)$

Therefore, the function is discontinuous.

7.

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} x^2 - x - 8 = (-2)^2 - (-2) - 8 = -2$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} -2x^2 - x + 4 = -2(-2)^2 - (-2) + 4 = -2$

$f(-2) = -2$

So $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$ $\quad \lim_{x\to -2^-} f(x) = f(-2)$ $\quad \lim_{x\to -2^+} f(x) = f(-2)$

Therefore, the function is continuous.

8.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x = 2(1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \sqrt{x^2 + 3} = \sqrt{(1)^2 + 3} = 2$

$f(1) = 2$

So $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ $\quad \lim_{x\to 1^-} f(x) = f(1)$ $\quad \lim_{x\to 1^+} f(x) = f(1)$

Therefore, the function is continuous.

9.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} -2x^2 + 2x + 13 = -2(3)^2 + 2(3) + 13 = 1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \sqrt{2x^2 - 3x - 9} = \sqrt{2(3)^2 - 3(3) - 9} = 0$

$f(3) = 1$

So $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$ $\quad \lim_{x\to 3^-} f(x) = f(3)$ $\quad \lim_{x\to 3^+} f(x) \neq f(3)$

Therefore, the function is discontinuous.

10.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{6}{x-3} + 3 = \frac{6}{(1)-3} + 3 = 0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -3x^2 - 2x + 4 = -3(1)^2 - 2(1) + 4 = -1$

$f(1) = 0$

So $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ $\quad \lim_{x\to 1^-} f(x) = f(1)$ $\quad \lim_{x\to 1^+} f(x) \neq f(1)$

Therefore, the function is discontinuous.

