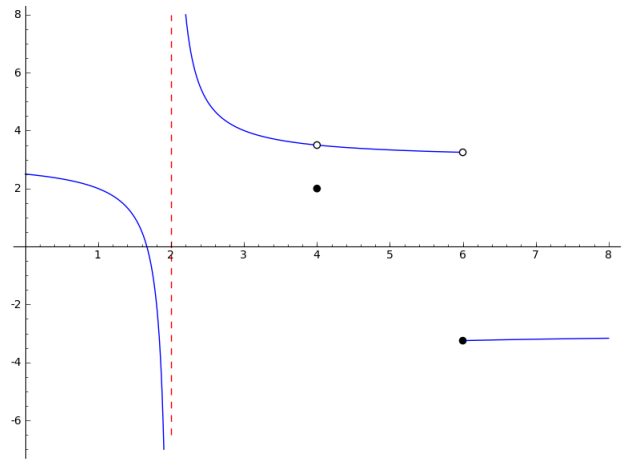
Practice problems set 4:
Continuity and discontinuity

1. Consider the following graph of the function $g(x)$.



   Classify the locations and types of discontinuities of $g(x)$ in the interval $(0, 8)$. Write the corresponding limits.

2. Determine the invervals on which the following functions are continuous.

   a) $f(x) = \dfrac{2x-6}{2x^2-x-3}$

   b) $g(x) = \sqrt{2-x}$

   c) $f(x) = \ln(1-x^2)$

   d) $g(x) = \dfrac{\sin(x)}{x-3}$

3. Classify the locations and types of discontinuities of the following functions. Write the corresponding limits.

   a) $f(x) = -\dfrac{1}{(x-1)^2}$

   b) $h(x) = \dfrac{3-\sqrt{x}}{9-x}$

   c) $g(x) = \begin{cases} x^2-2 & \text{if } x \neq -3 \\ 5 & \text{if } x = -3 \end{cases}$

   d) $f(x) = \begin{cases} e^x & x < 0 \\ x^2 & x \geq 0 \end{cases}$

Answers

1. Discontinuity: $x = 2$, Type: infinite, Corresponding limits: $\lim_{x \to 2^-} g(x) = -\infty$ and $\lim_{x \to 2^+} g(x) = +\infty$

   Discontinuity: $x = 4$, Type: removable, Corresponding limit: $\lim_{x \to 4} g(x) = 4$

   Discontinuity: $x = 6$, Type: jump (or step), Corresponding limits: $\lim_{x \to 6^-} g(x) = 3.5$ and $\lim_{x \to 6^+} g(x) = -3.5$

2. a) $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, +\infty\right)$

   b) $(-\infty, 2]$

   c) $(-1, 1)$

   d) $(-\infty, 3) \cup (3, +\infty)$

3. a) Discontinuity: $x = 1$, Type: Infinite, Corresponding limit: $\lim_{x \to 1} f(x) = -\infty$

   b) Discontinuity: $x = 9$, Type: Removable, Corresponding limit: $\lim_{x \to 3} h(x) = \dfrac{1}{6}$

   c) Discontinuity: $x = -3$, Type: Removable, Corresponding limit: $\lim_{x \to -3} g(x) = 7$

   d) Discontinuity: $x = 0$, Type: Jump, Corresponding limits: $\lim_{x \to 0^-} f(x) = 1$ and $\lim_{x \to 0^+} f(x) = 0$