

MTH 148

Solutions for Problems on the Intermediate Value Theorem

1. Use the Intermediate Value Theorem to show that there is a positive number c such that $c^2 = 2$.
2. If $f(x) = x^3 - x^2 + x$, show that there is $c \in \mathbb{R}$ such that $f(c) = 10$.
3. If $g(x) = x^5 - 2x^3 + x^2 + 2$, show that there is $c \in \mathbb{R}$ such that $g(c) = -1$.

In problems 4–7, use the Intermediate Value Theorem to show that there is a root of the given equation in the given interval.

4. $x^3 - 3x + 1 = 0$, $(0, 1)$
5. $x^5 - 2x^4 - x - 3 = 0$, $(2, 3)$
6. $x^3 + 2x = x^2 + 1$, $(0, 1)$
7. $x^2 = \sqrt{x+1}$, $(1, 2)$
8. Let f be a continuous function on $[0, 1]$. Show that if $-1 \leq f(x) \leq 1$ for all $x \in [0, 1]$ then there is $c \in [0, 1]$ such that $[f(c)]^2 = c$.