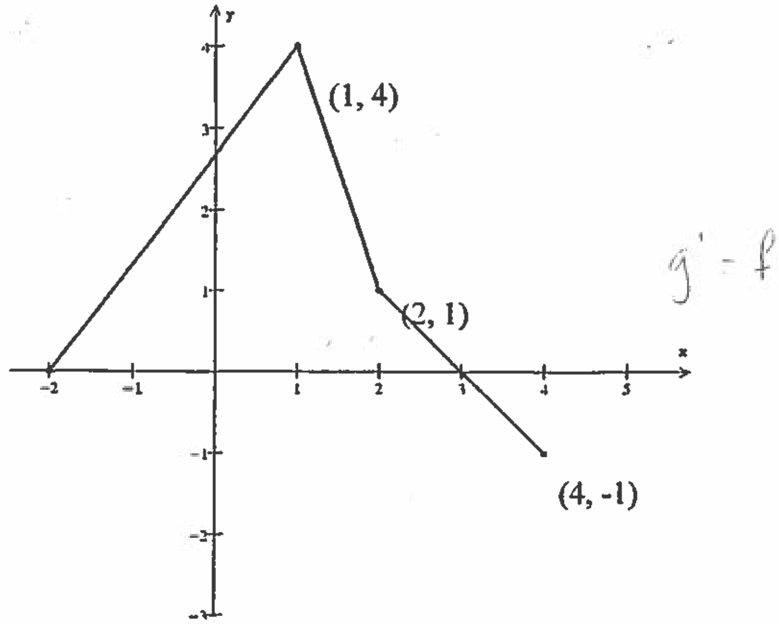


2. (Calculator) The graph of the function f, consisting of three line segments, is given above. Let $g(x)=\int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.
(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.
(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.
(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

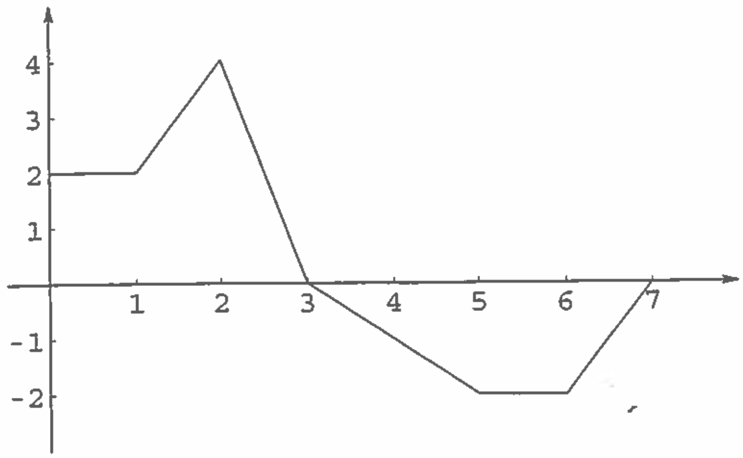
Example 6: Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(0), g(1), g(2), g(3)$, and $g(6)$.

(b) On what interval is g increasing?

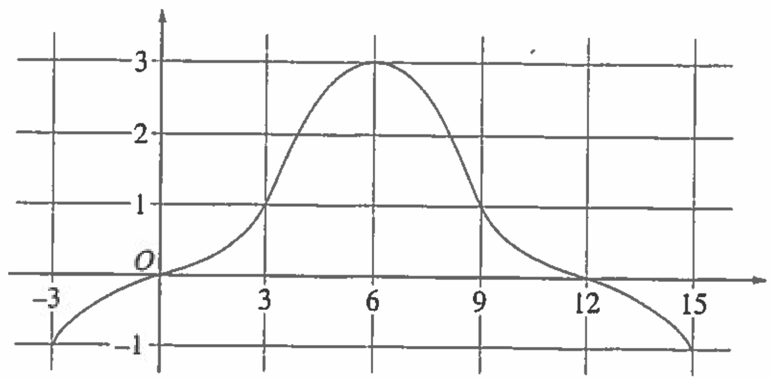
(c) Where does g have a maximum value?

(d) Sketch a rough graph of g.



CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

   (a) Find $g(6)$, $g'(6)$, and $g''(6)$.

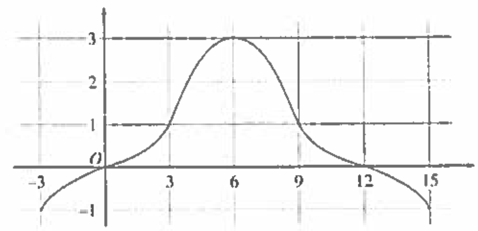
   (b) On what intervals is g decreasing? Justify your answer.

   (c) On what intervals is the graph of g concave down? Justify your answer.

   (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

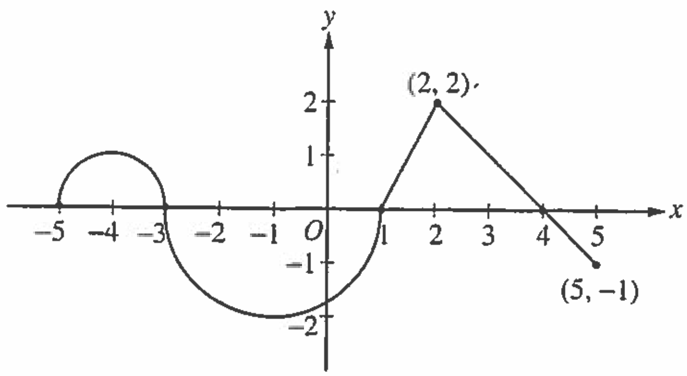
(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5$ $g'(6) = f(6) = 3$ $g''(6) = f'(6) = 0$	3 $\begin{cases} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{cases}$
(b) g is decreasing on $[-3,0]$ and $[12,15]$ since $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.	3 $\begin{cases} 1 : [-3,0] \\ 1 : [12,15] \\ 1 : \text{justification} \end{cases}$
(c) The graph of g is concave down on $(6,15)$ since $g' = f$ is decreasing on this interval.	2 $\begin{cases} 1 : \text{interval} \\ 1 : \text{justification} \end{cases}$
(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$ $= 12$	1 : trapezoidal method

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

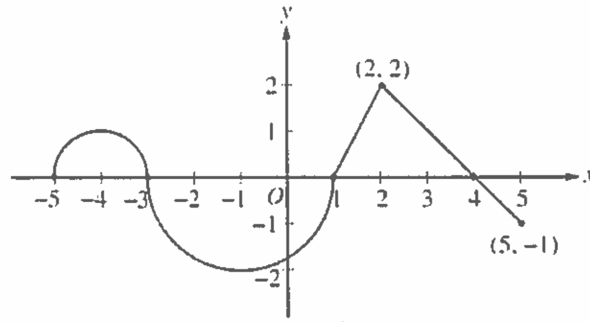
(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1$, and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1$, and 2.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

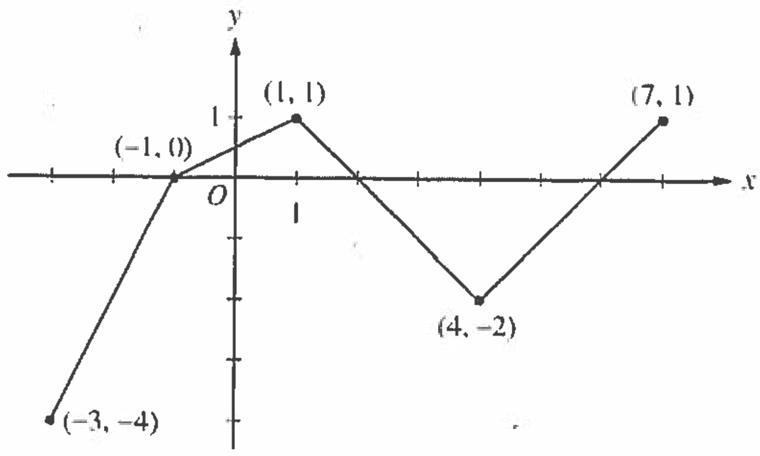
$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$



Graph of g'

NO calc

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

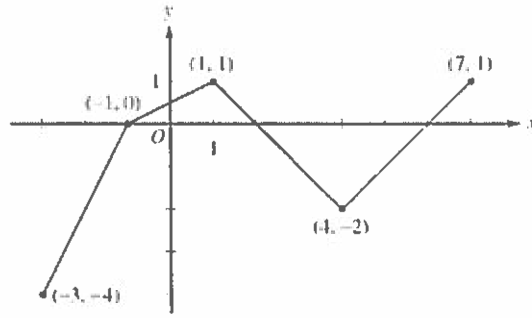
(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

Question 5



Graph of g'

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$