## 3.7 Problems of Motion

**734.** A car is moving along Highway 20 according to the given equation, where $x$ meters is the directed distance of the car from a given point $P$ at $t$ hours. Find the values of $t$ for which the car is moving to the right and when it is moving to the left. Draw a diagram to describe the motion of the car.

a) $x = 2t^3 + 15t^2 + 36t + 2$ b) $x = 2t^3 + 9t^2 - 60t - 7$

**735.** A car is moving along Highway 138 according to the given equation, where $x$ meters is the directed distance of the car from a given point $P$ at $t$ hours. Find the values of $t$ for which the acceleration is zero, and then find the position of the car at this time.

a) $x = \frac{1}{4}t^4 + \frac{1}{6}t^3 - t^2 + 1$ b) $x = -3\sqrt{t} - \dfrac{1}{12\sqrt{t}}$ for $t > 0$

**736.** A snail moves along the $x$-axis so that at time $t$ its position is given by $x(t) = 3\ln(2t-5)$, for $t > \frac{5}{2}$.

a) What is the position and the velocity of the snail at time $t = 3$?

b) When is the snail moving to the right, and when is it moving to the left?

**737.** An ant moves along the $x$-axis so that at time $t$ its position is given by $x(t) = 2\cos\left(\frac{\pi}{2}t^2\right)$, for values of $t$ in the interval $[-1, 1]$.

a) Find an expression for the velocity of the ant at any given time $t$.

b) Find an expression for the acceleration at any given time $t$.

c) Determine the values of $t$ for which the ant is moving to the right. Justify your answer.

d) Determine the values of $t$ for which the ant changes direction. Justify your answer.

**738.** A particle is moving along the $x$-axis so that its position is given by

$$x(t) = \frac{3\pi}{2}t^2 - \sin\left(\frac{3\pi}{2}t^2\right),$$

for $0 < t \leq 2$.

a) Find an expression for the velocity of the particle at any given time $t$.

b) Find an expression for the acceleration at any given time $t$.

c) Find the values of $t$ for which the particle is at rest.

d) Find the position of the particle at the time(s) found in part c).

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Thus metaphysics and mathematics are, among all the sciences that belong to reason, those in which imagination has the greatest role. I beg pardon of those delicate spirits who are detractors of mathematics for saying this .... The imagination in a mathematician who creates makes no less difference than in a poet who invents.... Of all the great men of antiquity, Archimedes may be the one who most deserves to be placed beside Homer. —*Jean le Rond d'Alembert*

**739.** At time $t \geq 0$, the velocity of a body moving along the $x$-axis is $v(t) = t^2 - 4t + 3$.

a) Find the body's acceleration each time the velocity is zero.

b) When is the body moving forward? Backward?

c) When is the body's velocity increasing? Decreasing?

**740.** The position of a ball moving along a straight line is given by $s(t) = \frac{4}{3}e^{3t} - 8t$.

a) Write an expression for the velocity at any given time $t$.

b) Write an expression for the acceleration at any given time $t$.

c) Find the values of $t$ for which the ball is at rest.

d) Find the position of the ball at the time(s) found in part c).

**741.** A racehorse is running a 10 furlong race (1 furlong is 220 yards). As the horse passes each furlong marker, $F$, a steward records the time elapsed, $t$, since the beginning of the race, as shown in the table below.

| $F$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0 | 20 | 33 | 46 | 59 | 73 | 86 | 100 | 112 | 124 | 135 |

a) How long does it take the horse to finish the race?

b) What is the average speed of the horse over the the first 5 furlongs?

c) What is the approximate speed of the horse as it passes the 3-furlong marker?

d) During which portion of the race is the horse running the fastest? Accelerating the fastest?

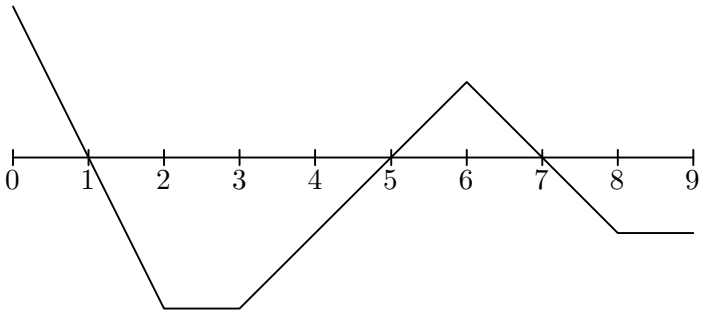
**742.** The graph below shows the velocity $v = f(t)$ of a particle moving on a coordinate line.

a) When does the particle move forward? move backward? speed up? slow down?

b) When is the particle's acceleration positive? negative? zero?

c) When does the particle move at its greatest speed?

d) When does the particle stand still for more than an instant?




Every job is a little self-portrait of the person who did it. —*Anonymous*

**726.** (a) $\{x|x \neq \pm 3\}$ (b) 0 (c) 0 (d) min at $(0,0)$ (e) inc for $x < -3$ and $-3 < x < 0$, dec for $0 < x < 3$ and $x > 3$ (f) none (g) ccup for $x < -3$ and $x > 3$, ccdown for $-3 < x < 3$

**729.** (a) $\{x|x > 0\}$ (b) 1 (c) none (d) max at $(e, \frac{1}{e})$ (e) inc for $0 < x < e$, dec for $x > e$ (f) $\left(e^{3/2}, \frac{3}{2e^{3/2}}\right)$ (g) ccup for $x > e^{3/2}$, ccdown for $0 < x < e^{3/2}$

**732.** 3.84

**733.** (a) mins at $x = -2.5$ and $x = 2$, max at $x = 0$ (b) ccup for $-3 < x < -1$ and $1 < x < 3$, ccdown for $-1 < x < 1$ and $3 < x < 4$

**735.** (a) at $t = \frac{2}{3}$ $x = \frac{53}{81}$, at $t = -1$ $x = \frac{1}{12}$ (b) at $t = \frac{1}{12}$ $x = -\frac{2}{3}\sqrt{3}$

**736.** (a) 0 (b) 6 (c) always right

**737.** (a) $v(t) = -2\pi t \sin(\frac{\pi}{2}t^2)$ (b) $a(t) = -2\pi(\sin(\frac{\pi}{2}t^2) + \pi t^2 \cos(\frac{\pi}{2}t^2))$ (c) right for $-1 < t < 0$, left for $0 < t < 1$ (d) 0

**738.** (a) $3\pi t - 3\pi t \cos(\frac{3\pi}{2}t^2)$ (b) $3\pi - 2\pi \cos(\frac{3\pi}{2}t^2) + 9\pi^2 t^2 \sin(\frac{3\pi}{2}t^2)$ (c) 0, $\sqrt{\frac{4}{3}}$, $\sqrt{\frac{8}{3}}$ (d) 0, $2\pi$, $4\pi$

**740.** (a) $4e^{3t} - 8$ (b) $12e^{3t}$ (c) $\frac{1}{3}\ln 2$ (d) $\frac{8}{3}(1 - \ln 2)$

**741.** (a) 135 sec (b) $\frac{5}{73}$ furlongs (c) $\frac{1}{13}$ furl/sec (d) the last and first furlong

**746.** one piece 14.8 m, other 15.2 m; use all iron to make the triangle

**747.** $8 \times 8 \times 4$ cm

**748.** 42

**749.** $225 \times 150$ m

**750.** \$2.95

**751.** $\frac{\pi}{4}$

**752.** (a) $\approx 578.7$ cm$^3$ (b) $616\frac{2}{3}$ cm$^2$

**753.** (a) $[0, B]$, max dosage, scale factor (b) $\frac{2}{3}B$ (c) $\frac{4}{27}AB^3$ (d) $\frac{1}{3}B$

**754.** $R^2$

**755.** 10 shipments of 240 players each

**756.** $\frac{1}{\sqrt{2e}}$

**757.** (a) $\frac{\pi}{3}, \frac{4\pi}{3}$ (b) 1 (c) $\frac{\pi}{3}, \frac{4\pi}{3}$

**759.** $-1$

**769.** crit pt is $x = 1$, inc for $x < 1$, dec for $x > 1$, extrema at $x = 1$

**778.** (c) $\arctan x + \dfrac{x}{1+x^2}$
(e) $-25x^{-2} + 6x^{-1/2}$
(f) $30x^4 - 60x^3 + 20x - 21$
(g) $\dfrac{-2(x^2+1)}{(x^2-1)^2}$

**780.** $y = \frac{1}{2}t$

**781.** (d) $y = e^x$

**782.** $y' = \cot x$

**783.** $e^3$

**788.** $\frac{\pi}{3}$

**789.** (a) 4, 0 (b) $-1$, $-1$, 1, $\frac{1}{2}$ (c) 0, $-\frac{3}{2}$

**790.** (a) 1, $\frac{3}{4}$ (b) positive (c) zero

**791.** (a) $h' = 0$ (b) $k' = 0$

**793.** $\frac{1}{4}$

**794.** (a) odd
(b) $\dfrac{1 + \cos x + x \sin x}{\cos^2 x}$
(c) $y = 2x$

**795.** A

**796.** E

**798.** (a) max at $x = -1$, mins at $x = \pm 3$ (b) $x = 0$, $x = 1$

**799.** (b) $x = 0$ (c) everywhere

**800.** $\mathbb{R}$, min at $(0, \frac{1}{10})$

**801.** $\mathbb{R}$, max at $(0, 10)$

**803.** $\{x|x \neq -1\}$, no extrema

**804.** $\{x|x > 0\}$, no extrema

**806.** $e^{-x}(x-2)$

**807.** $e^x(x^2 + 4x + 2)$

**808.** $e^{x+e^x}(1 + e^x)$

**810.** (a) $\dfrac{-2xy}{x^2+y^2}$ (b) $y = \frac{4}{5}x - \frac{13}{5}$ (c) $\sqrt[3]{-13}$

**811.** (a) 0, $\frac{\pi}{2}$, $\pi$ (b) $\frac{\pi}{6} < x < \frac{\pi}{2}$ and $\frac{5\pi}{6} < x < \frac{3\pi}{2}$ (c) min of $-\frac{1}{4}$, max of 2

**812.** (a) $y = 4x + 2$ and $y = 4x - 2$ (b) 1 (c) 0

**813.** (a) $x = -2$ (b) $x = 4$ (c) $-1 < x < 1$ and $3 < x < 5$

**814.** (a) $\{x|x \neq 0\}$ (b) even (c) maxs at $x = \pm 1$ (d) $f(x) \leq \ln \frac{1}{2}$

**815.** (b) $c \approx 1.579$ (c) $y \approx 1.457x - 1.075$ (d) $y \approx 1.457x - 1.579$

**817.** (a) $k = -2$, $p = 2$ (b) always inc (c) $(1, 1)$

**818.** (a) min of $\dfrac{-e^{5\pi/4}}{\sqrt{2}}$, max of $e^{2\pi}$ (b) inc for $0 < x < \frac{\pi}{4}$ and $\frac{5\pi}{4} < x < 2\pi$ (c) $\pi$

**819.** (a) 100 (b) $y = \frac{3}{5}x + 20$ (c) yes, the top 5 ft of the tree

**820.** C

**821.** B

**822.** B

**823.** A

**824.** D

**825.** C

**826.** D

**827.** C

**828.** D

**829.** E

**830.** B

**831.** E