UNIT 5 - Series

Name ____________________________________ PER_____ DATE_________

DO NOW

Answer the following question and show your work?

What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1
(B) 2
(C) 4
(D) 6
(E) The series diverges

TPS-C

If $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$, then $f'(x) =$

(A) $\frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} + \frac{x^7}{7 \cdot 3!} + \frac{x^9}{9 \cdot 4!} + \cdots + \frac{x^{(2n+1)}}{(2n+1)n!} + \cdots$

(B) $x + \frac{3x^3}{2!} + \frac{5x^5}{3!} + \frac{7x^7}{4!} + \cdots + \frac{(2n-1)x^{(2n-1)}}{n!} + \cdots$

(C) $2 + 2x^2 + x^4 + \frac{x^6}{3} + \cdots + \frac{2x^{2(n-1)}}{(n-1)!} + \cdots$

(D) $2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{(2n-1)}}{n!} + \cdots$

Taylor Polynomials and Approximations

Polynomial functions can be used to approximate other elementary functions such as $\sin x$, e^x, and $\ln x$.

Example 1:

Find the equation of the tangent line for $f(x) = \sin x$ at $x = 0$, then use it to approximate $\sin(0.2)$. Is this an over or an under approximation of $\sin(0.2)$?

The equation of the tangent line used in Ex. 1 is called a **first-degree Taylor polynomial**. Taylor polynomials of higher degree can be used to obtain increasingly better approximations of non-polynomial functions within a certain **radius** from a **center of approximation** $x = c$.

Example 2:

On your calculator graph $y1 = \sin x$. Use the following window: $X[-9,9]$, $Y[-4,4]$. Now in $y2 =$, graph successively, adding an extra term each time, the following: $y2 : x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

What do you notice? What is $y1(0)$? $y2(0)$? What is $y1(0.2)$? $y2(0.2)$?

Definition of an *n*th-degree Taylor polynomial:

If f has n derivatives at $x = c$, then the polynomial

is called the *n*th-degree Taylor polynomial for f centered at c, named after Brook Taylor, an English mathematician.

Note 1:

Note 2:

*n*th-degree Maclaurin polynomial for f, named after Scottish mathematician, Colin Maclaurin.

Note 3:

Note 4:

Example 1

Construct the seventh order Taylor polynomial and the Taylor series for sin x and x = 0.

Example 2

Find the fourth order Taylor polynomial that approximates y = cos 2x, near x = 0.

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Examples

List the first four non-zero terms of the Maclaurin Polynomials for $f(x)=\sin x$, $f(x)=\cos x$, and $f(x)=e^{x}$, the find the following Maclaurin Polynomials.

(a) $g(x)=\sin(2x)$ (b) $g(x)=x\cos(x)$ (c) $g(x)=4e^{x^2}$